AN INVESTIGATION OF THE STABILITY OF A SYSTEM OF TWO SHIPS EMPLOYING AUTOMATIC CONTROL WHILE ON PARALLEL COURSES IN CLOSE PROXIMITY

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by

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ABSTRACT

AN INVESTIGATION OF THE STABILITY OF A SYSTEM OF TWO SHIPS EMPLOYING AUTOMATIC CONTROL WHILE ON PARALLEL COURSES IN CLOSE PROXIMITY

Charles Natale Calvano

Submitted to the Department of Naval Architecture and Marine Engineering on 21 May 1970 in partial fulfillment of the requirements for the degrees of Master of Science in Ocean Engineering and Naval Engineer.

An introduction outlining the development of ship operations in close proximity for underway replenishment is followed by a survey of the literature dealing with the measurement of interaction effects between ships under such circumstances. A brief outline of the theoretical treatment of ship motions in open water is given and followed by the modifications to the theory required for treatment of the two ship system. The applicable equations are written for calm water and suitably linearized. The techniques of treating the differential equations as algebraic equations, and of solving the equations by determinant evaluation are described.

Extension of available interaction and open water data for two different ships to a full data set for a single, modified ship is carried out, and the numerical values of resulting hydrodynamic derivatives are given. The numerical solution of the equations and the extraction of stability roots are then described and results for 91 typical trial data sets are listed. A discussion of the results obtained is followed by an evoluation of the correctness of the solution method employed.

The computer programs used throughout are described in detail in four appendices.

Thesis Supervisor: Martin A. Abkowitz
Title: Professor of Naval Architecture

ACKNOWLEDGEMENT

Thesis acknowledgements traditionally single out the thesis supervisor as the primary source of aid to the author. This tradition is doubly applicable to this thesis, for reasons other than the usual ones. First, Professor Martin A. Abkowitz is largely responsible for previous work in the field of ship motion which made possible the mathematical approach used herein, and one need only refer to Chapter II to determine the extent of the author's reliance on his efforts. Second, he greatly enhanced the effectiveness of this thesis effort in fulfilling its function as a learning process, by his adherence to the principle that the thesis should "comprise an original investigation.....by a single student", (as it is supposed to), and by refraining from constant "second guessing" of the author's work.

As a result, of course, the errors inevitably included herein, remain the "property" of the author alone.

My wife, Charlotte, through her hours of listening to discussion of a subject with a limited range of interest, contributed significantly to the timely completion of the project.

Mr. David Burke, a fellow graduate student, who on at least two occasions helped me to see the forest when the trees had obscured my view, also deserves the author's thanks.

And Miss Sandra Vivolo, secretary in the Department of Naval Architecture and Marine Engineering, patiently endured the never-ending changes and queries of "is that chapter finished yet?", that any typist knows so well.

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Chapter I

INTRODUCTION

I. Background-Replenishment at Sea

In their efforts to achieve a degree of independence from land bases, the navies of the world have, for decades, practiced various methods of underway refueling and replenishment of their ships. As techniques were improved and experience was gained, these operations progressed from the status of infrequent, makeshift affairs, to that of routine evolutions. These replenishment methods have ranged from actually lashing together the supply ship and receiving ship while passing coal and provisions from one to the other, (one assumes this was attempted in only the very calmest of seas, or in sheltered roadsteads), to the transfer of cargo via "vertical replenishment" (VERTREP) helicopters. Other methods employed over the years have included the astern refueling method and the alongside method for transferring fuel and cargo.

In the former, the receiving ship recovers a fuel hose trailed in the water by the tanker and takes on fuel while steaming in the tanker's wake. In the alongside method, the receiving ship overtakes the supply ship on a parallel course, laterally separated by from 40 to 180 feet. The transfer is effected while the ships remain abreast at this distance.

The alongside method is the standard of the U.S. Navy, (augmented by VERTREP for cargo transfer, though not, of course, for refueling), and such replenishment operations

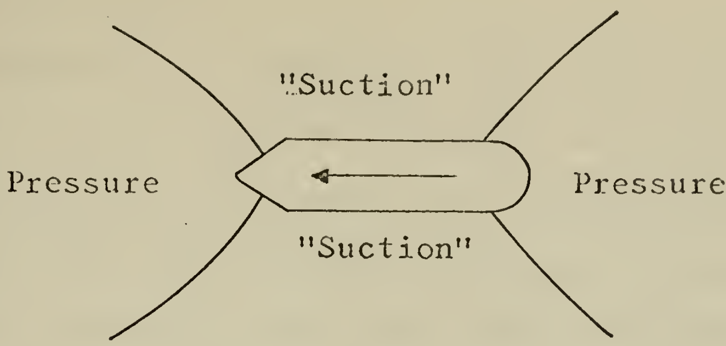
have become almost a way of life for U.S. combatant ships. Unless, and until, all U.S. combatant ships become nuclear fueled, this method will probably remain in wide use.

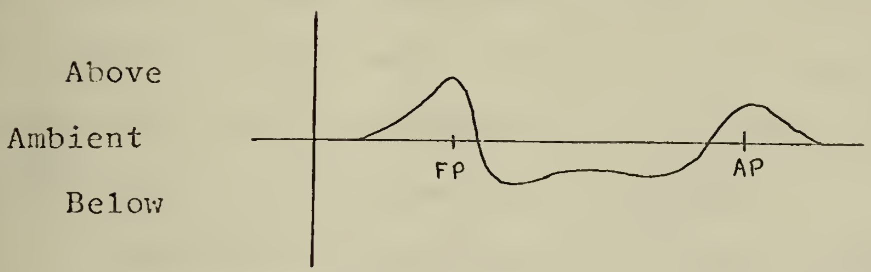
Of course there are other situations than replenishment when two ships might wish to operate in close alongside proximity while underway. To generalize the treatment herein, the forces and moments arising from the tension in the replenishment rig lines is not included. Thus the situation occurring whenever two ships must steam alongside each other at close quarters is that which is treated in this thesis.

II. Some Problems

Although alongside replenishment has become commonplace, this type of close-in steaming has not ceased to be a dangerous undertaking. There are, of course, dangers arising from the lack of freedom to maneuver to avoid other shipping, or, in time of war, the actions of an enemy. But the ever-present source of danger to two ships steaming close together on parallel courses is this very proximity of the ships, causing each to constitute a collision hazard for the other.

While alongside, the ships must make almost constant rudder and engine speed adjustments to maintain position. Because the ships are close, the presence of each affects the forces and moments on the other through the influence of each on the pressure field and flow patterns around the other. (See Figure I-1). Thus, neither ship will respond as it does in open water where these influences are not present. And





Pressure Field about a Ship in Open Water

(From Reference A-1)

FIGURE I-1

as the ships alter their relative positions, the direction and magnitude of these influences are continuously altered.

The repeated corrections required over a period of up to several hours, introduce hundreds of opportunities for human error or momentary lack of attention to give rise to damaging consequences in the form of a collision. Any method or device which could serve to reduce the probability of such errors occurring would serve to lessen a primary hazard for two ships operating in this way.

III. Automatic Control

The introduction of automatic rudder angle and engine speed control to the alongside operation is an attractive prospect. Such a control system would employ sensors to measure longitudinal and lateral separation of reference points on the two ships, (and their rates of change), and compare these values to desired equilibrium values. It would then generate rudder and engine orders to reduce the deviations to acceptable magnitudes. Such a system could be a boon to conning officers and helmsmen by relieving them of much of the tedium of maintaining position alongside.

A theoretical investigation of such a system is the subject of this thesis. The problem will be approached from a mathematical standpoint as a two-body system of ship motions. The hardware required to make it operable will not be examined or described. It will be assumed that a sensor, (doppler sonar, for instance), is available to continually measure lateral and longitudinal ship separations and their rates of change.

Once the deviation from desired ship separation and heading is known, an automatic control system can order rudder and engine speed changes to reduce the deviations. The various constants associated with a theoretical control system meeting this description will be a part of the mathematical model of the two ship system. The stability of the system will be investigated, to determine if it appears feasible to employ such an automatic control system to keep the ships in position alongside. (By stability is meant the tendency of the two

ship system to return to an equilibrium condition after receiving a small disturbance from that equilibrium condition. If the system "returns, or tends to return, to the equilibrium condition when the disturbance is removed, then it is stable". [A-3]*)

IV. Historical Aspects--The Literature

A. Taylor.

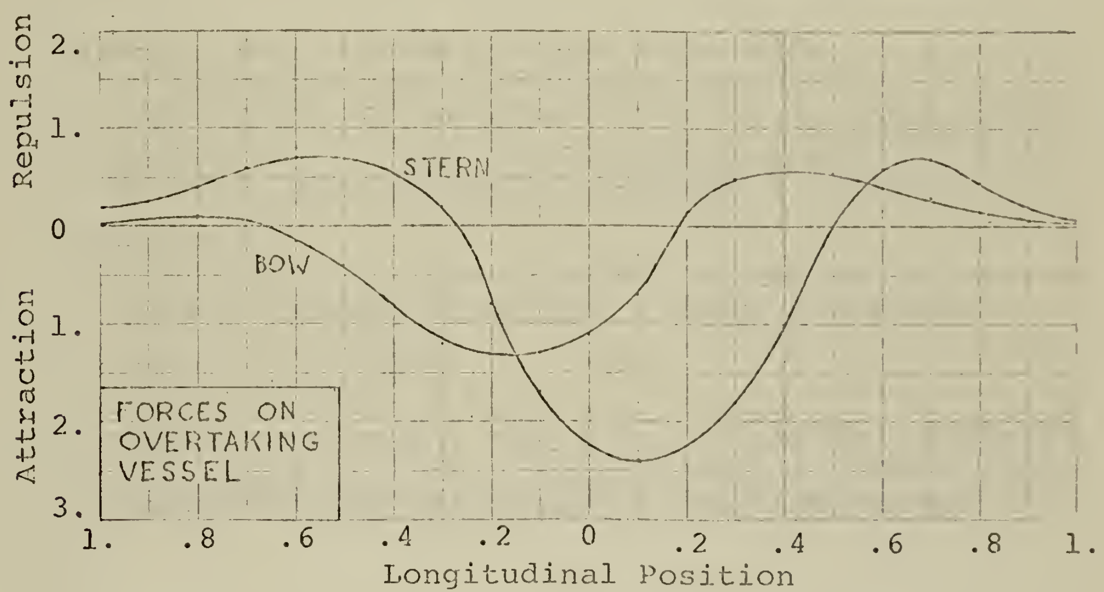
As far as the author has been able to determine, the first systematic attempt to measure interaction phenomena for two ships on parallel courses in open water was made by David W. Taylor, the ubiquitous Naval Constructor. He conducted a series of tests utilizing four models, each of 3,000 pounds displacement, at the Model Basin in Washington, in 1909. [A-2]

Taylor's tests were conducted by towing the models in pairs, either abreast of one another, or with one model a fixed distance ahead of the other. The models were not self-propelled, and did not have propellers affixed. They were towed under towing carriages, with their bows and sterns prevented from translating laterally, but free to move vertically, changing draft and trim while being towed. Water depths were "many times the draught of the models". Attraction and repulsion forces were measured at the attachment points near the bow and stern of each model. Table I-1 lists the characteristics of the models employed and Figure I-2 is a typical plot of the results Taylor obtained.

*Letter-number citations in brackets refer to the references listed at the end of the body of this thesis.

Model #	Beam (B) ft.	Draft (H) ft.	$\frac{B}{H}$	Prismatic Coeff C_p	Midships Coeff C_m	Block Coeff C_b
834	3.692	1.263	.560	.560	.900	.504
838	3.503	1.198	2.92	.560	1.00	.560
858	3.586	.957	3.75	.740	.926	.685
866	2.778	1.235	2.25	.740	.926	.685

TABLE I-1



Side Force in Units of Resistance

vs.

Longitudinal Position

Figure I-2

In conducting his tests, Taylor found that he had difficulty towing the models on a straight course. In view of the fact that towing carriages were used, this is hard to understand, but was explained no further. As a result of this difficulty, he felt that the forces measured could not be regarded as highly accurate, but felt that they showed the tendencies and general nature of the interaction, or, to use his term, "suction", phenomena.

The sequence of events, when vessel B overtakes vessel A on a parallel course, seemed to be about as follows:

1. As B's bow overlaps A's stern, a small repulsion force is felt at both B's bow and stern.
2. As B continues to overtake A, B's bow is attracted, while its stern is repelled, causing B to tend to turn into A.
3. As B draws abreast of A, both B's bow and stern are drawn in toward A.
4. As B pulls up farther, the force on its bow switches to repulsion, while its stern is still attracted to A.
5. As B draws farther forward, to where its stern is approximately abreast of A's midlength, both B's bow and stern are repelled.

Taylor noted further, that the occurrence of these events was somewhat more consistent with the "finer" models, while the results with the "full" models tended to be erratic. The effect of the "suction" forces on the models' resistance was

not measured, but Taylor did note that the side forces "varied with speed as the resistance of the model"; that is the interaction forces increased as the square of the velocity.

It is evident from the discussions accompanying Taylor's paper that there was disagreement within the Naval Architect community concerning the significance and, in fact, even the existence, of interaction forces for full scale ships.

B. Gibson and Thompson.

In 1913, Gibson and Thompson conducted a series of full scale tests with small steam launches in the Thames River near London, to verify the existence, and measure the effect of interaction phenomena. [B-2] They did note effects similar in type to those described by Taylor, but due to instrumentation difficulties, their report contains no quantitative information of a useful nature in considering interaction effects for ships.

C. Robb.

A disastrous collision between the liner QUEEN MARY and the destroyer H.M.S. CURACOA in 1942 led to a series of experiments reported by A.M. Robb in 1949. [A-8] In these tests, the models used were of these two ships and made to a 1:56 scale. This resulted in the QUEEN MARY model being "rather more than 18 feet" in length, and that of the CURACOA "rather more than 8 feet". In these experiments, both models were self-propelled, with first one, then the other, being constrained to run on a straight course while the other passed

on a nearly parallel course. Robb again confirmed the existence of significant interaction effects drawing the smaller ship into the larger, in some circumstances, and his article is accompanied by several sketches showing the trajectories of the ships for various speeds and lateral separations. The results presented are largely qualitative in nature, and do not disagree with those obtained by Taylor 40 years earlier.

But again, in the discussions of Robb's paper, the fact that the very existence of interaction effects was doubted in some quarters was brought out. [B-5] Professor Prohaska, in his discussion, stated, "The question of interaction between passing vessels has always given rise to interesting exchanges of opinion.....a great many experts, amongst whom are numerous seafaring people with great experience, deny the existence of interaction, or at least think it without importance in unrestricted water." (Prohaska, however, did not share this view.)

D. Restricted Water Studies.

Over the years, there have been numerous other experiments and theoretical developments concerning the interaction between a ship and a stationary, constant-geometry object, such as a canal wall or bank. References A-6, A-10, B-6, B-10, B-11, B-12, B-13 and B-18 all deal with this phenomenon. A ship, being a large object, can appear, in its effects, as a wall would appear to another ship passing near. But of course, the geometry of the ship-object is different than when the object is a wall. And, also, neither ship is immovable, as is the wall.

Most of the investigations concerning interaction in canals or channels include shallow water conditions. The shallow water tends to emphasize the interaction effects caused by the wall. This investigation, though, is concerned with the interaction between ships in deep water.

E. Newton.

The single study of interaction effects which proved to be of greatest use to the author in attempting to obtain quantitative information concerning the interaction forces and moments was that listed as reference A-1, by R.N. Newton and reported at the First Symposium on Ship Maneuverability conducted at the David Taylor Model Basin in 1960.

Newton reported on experiments conducted at the Admiralty Experiment Works (AEW) in England, between 1946 and 1948 and which were inspired by the growth of alongside replenishment experienced during the war. In his paper, Newton acknowledges the pioneering work of Taylor in this field and laments the near-total lack of work in this area in the years between Taylor's tests and the AEW efforts, although he noted the wealth of information available to him concerning interaction effects in restricted channels and shallow water.

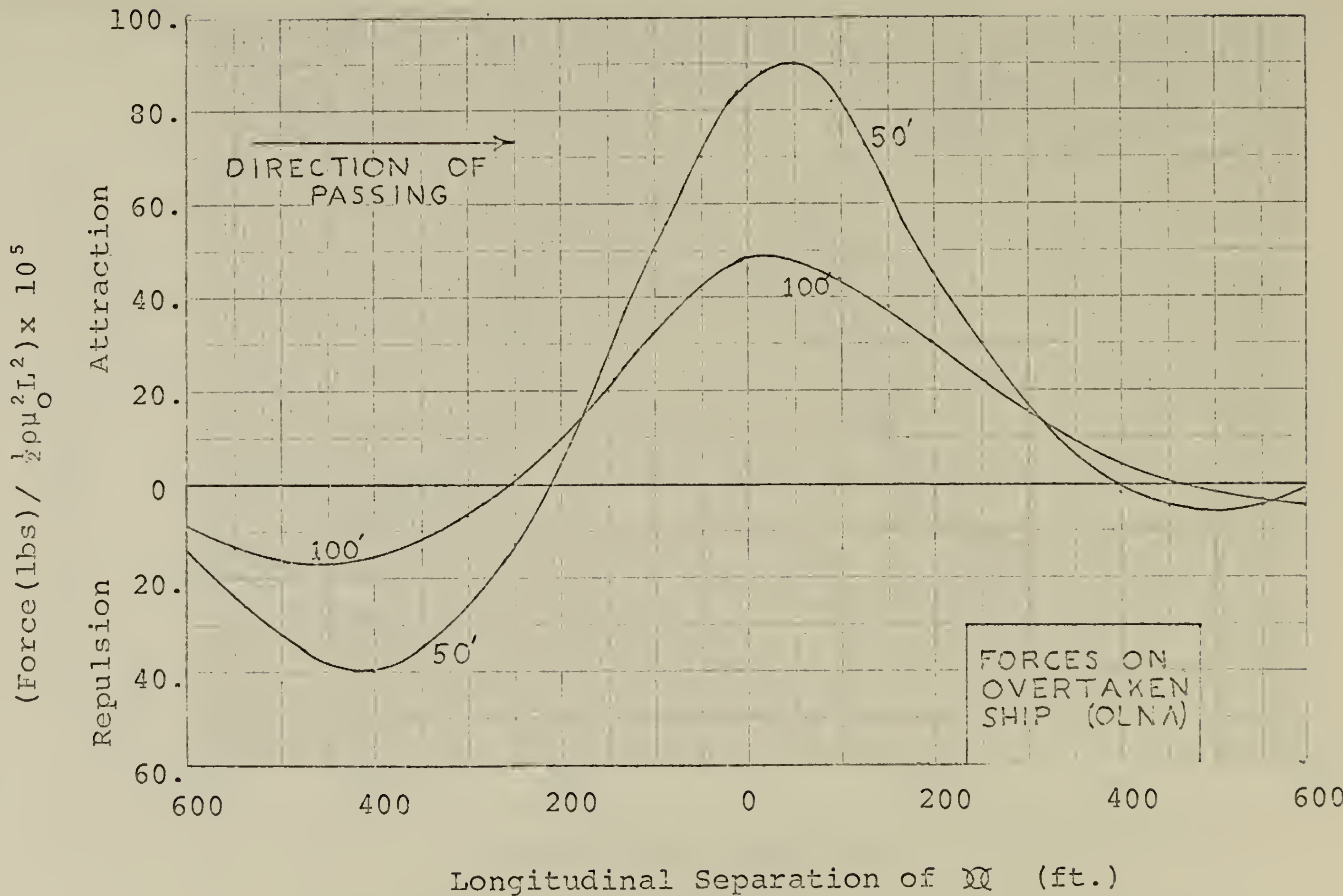
Newton's experiments were with models of a battleship, H.M.S. KING GEORGE V and a replenishment supply ship, R.F.A. OLNA. (See Table I-2 for ship characteristics.) The tendencies exhibited by the models were confirmed by full scale trials at sea.

	K.G.V	OLNA
Length on W.L., ft.	740	547
Beam, ft.	103	70
Draft, ft.	29.3	30
Displacement, tons	39,890	23,570
Block Coefficient	.611	.714
Corresponding water depth	75 fathoms	

TABLE I-2

Tests were conducted with both towed and self-propelled versions of the models. The models were allowed complete vertical freedom, but constrained from lateral motion. Forces and moments were measured at longitudinal separations of the midships sections of the ships from 600 ft. aft of the abreast position to 600 ft. forward of this position, at two lateral separations; 50 and 100 ft. side to side distance. With the ships in the abreast position, the side to side distance was varied from 25 to 140 ft. (All distances are the corresponding full scale values.)

Figures I-3 and I-4 show the longitudinal variation of the forces and moments experienced by OLNA at the two lateral separations and Figure I-5 shows the lateral variation with the ships abreast. These data formed the basis for quantitative interaction forces and moments used in this investigation, as OLNA is not radically different in hull characteristics from the MARINER class vessel used for this study. In the figures, OLNA is the ship being overtaken. The agreement with the results obtained by Taylor is apparent.



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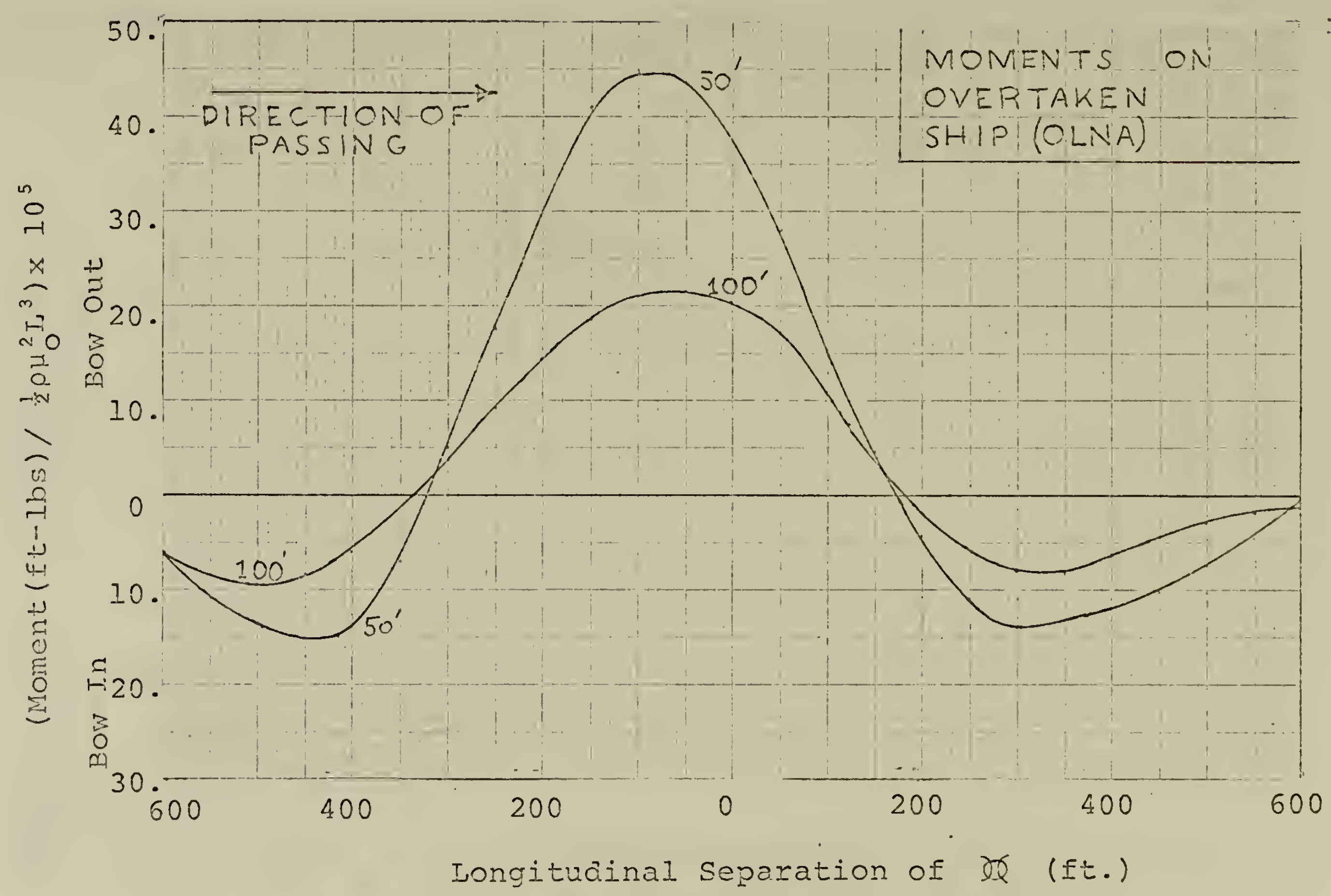
Non-Dimensional Force vs. Longitudinal Separation for Side-to-Side Separations of 50 and 100 feet

Figure I-3

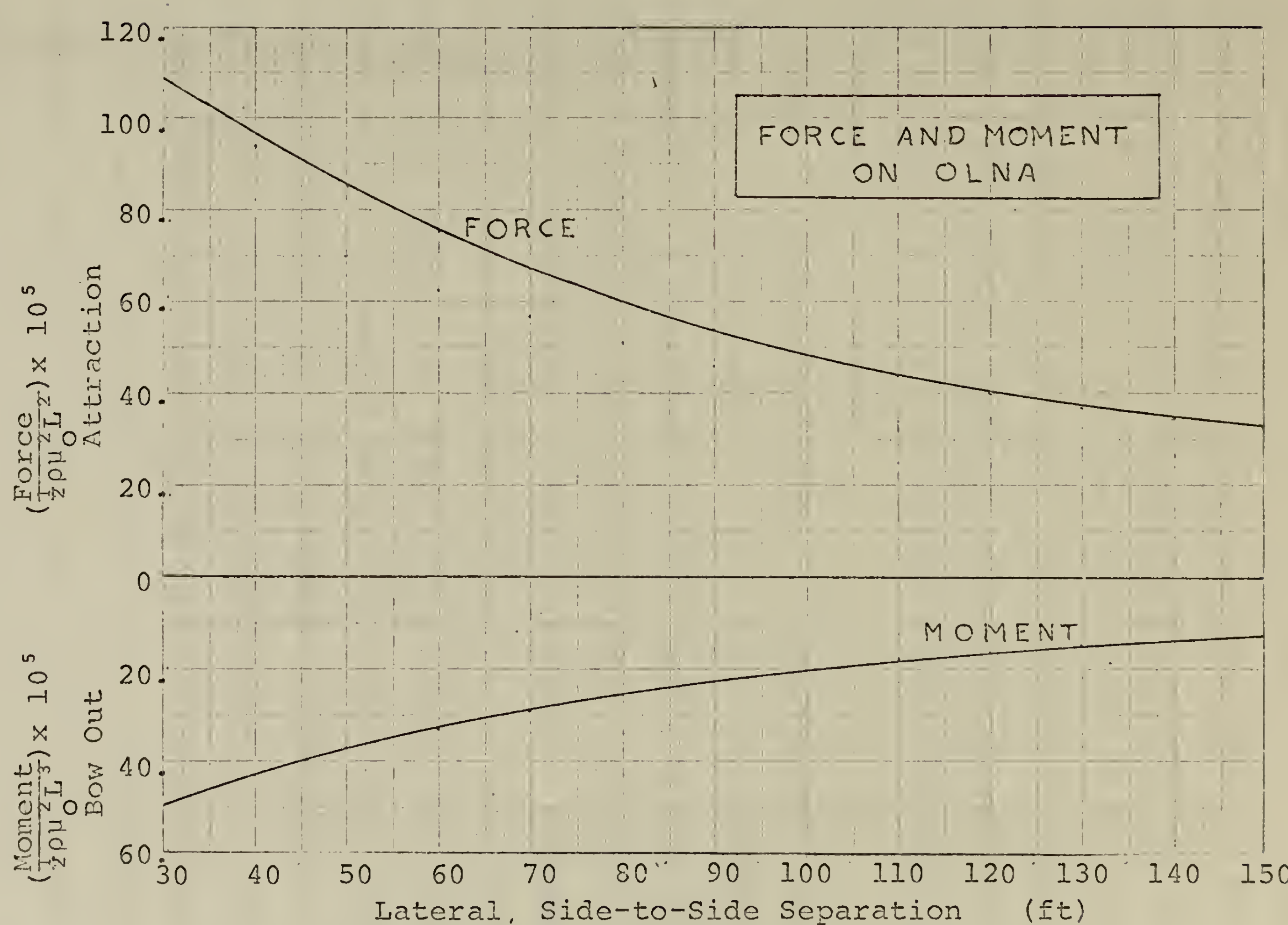


Non-Dimensional Moment vs. Longitudinal Separation

for Side-to-Side Separations of

50 and 100 feet

Figure I-4



Non-Dimensional Force and Moment
vs.
Lateral Separation
at 0-ft. Longitudinal Separation

Figure I-5

F. Strom-Tejsen and Chislett.

In making a theoretical investigation such as this one, it is necessary to have realistic values for the hydrodynamic derivatives employed in the mathematical model. (The use of these derivatives, and the mathematical model are discussed in Chapter II, THEORY.) For two ships steaming in proximity, values of both the open water and interaction derivatives are needed, as forces and moments which are always present for a ship in open water will be present along with those caused by the interaction phenomena.

Thus, in this investigation it was necessary to find sources for typical values of both kinds of hydrodynamic derivatives. As mentioned earlier, Newton's work with KING GEORGE V and OLNA was the only available source of quantitative interaction information. It was, therefore, necessary to find values of the open water derivatives for a ship similar to either KING GEORGE V or OLNA. Strom-Tejsen and Chislett in reference A-5 determined most of the open water hydrodynamic derivatives of interest for a MARINER class merchant ship. (See Table I-3 for MARINER characteristics.) In reference A-4, Strom-Tejsen used these derivatives to theoretically predict the open water maneuvering behavior of a MARINER, and subsequent sea trials proved these values to be quite accurate. Fortunately, OLNA and the MARINER class hulls are sufficiently similar to expect the hydrodynamic derivatives of one to reasonably resemble those of the other. Therefore, in the development of the results of this thesis, the open water

derivatives of the MARINER were used, and, to account for interaction effects, derivatives derived from the interaction force and moment data of the OLNA experiment were added to these to obtain the entire list of needed derivatives. Thus, the ship type investigated is a somewhat modified MARINER class ship.

MARINER Characteristics	
Length, ft.	528.5
Beam, ft.	76.0
Draft, ft.	29.75
Displacement, tons	20,900
Block Coefficient C_b	.6125
Prismatic Coeff. C_p	.6246
Midships Section Coeff. C_m	.9807

TABLE I-3

Chapter II

THEORY

I. Background

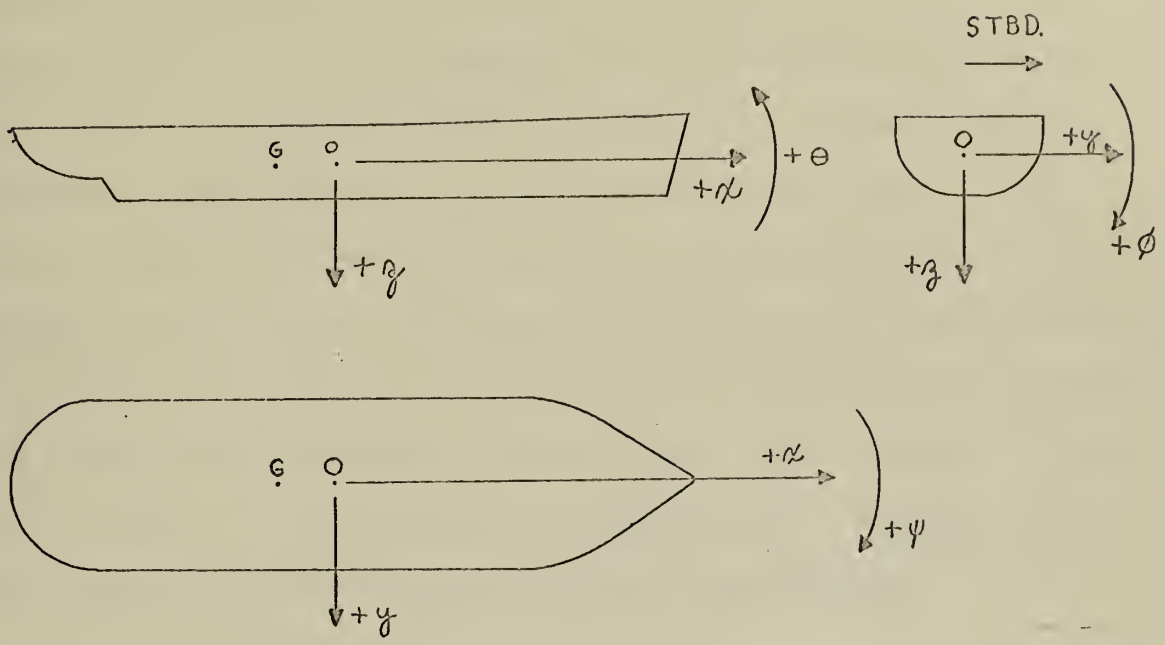
The derivation of a mathematical model representing the steering and maneuvering of a surface ship in open water is given by Abkowitz in reference A-3. In its general form, the model accounts for non-linear, as well as linear, effects, and in his computerized prediction of open water, surface ship maneuvers, Strom-Tejsen uses the Abkowitz model with up to third order terms. [A-4]

This mathematical model, in linearized form, serves as the basis for the analysis made in this thesis, with appropriate extension of the model to include the interaction effects which are not present in open water, but which are important for two ships on parallel courses in close proximity. The model used herein also is modified to represent the two-ship system, rather than the motions of a single ship, and accounts for the fact that both ships are free to move in the horizontal plane. (For a more complete description than the outline which follows, of the derivation of the open water model, reference A-3 is suggested).

II. Reference Axes

The reference axes used constitute an orthogonal, right-handed system which takes advantage of the existence of a vertical plane of symmetry through the centerline of the ship. The selection of the axis system illustrated introduces

simplifications in the equations of motion by virtue of the fact that the plane of symmetry of each ship is parallel to the principal axes of inertia of the ship.



(See Table II-1 for definitions of all symbols used in this chapter)

Reference Axis System

FIGURE II-1

III. Equations of Motion for a Ship Moving in the Horizontal Plane

A general form of the equations of motion for a body which is allowed to move in all the six degrees of freedom is given by Abkowitz.

Normally, when dealing with steering and maneuvering of surface ships in open water, the primary motions are considered to take place in the horizontal plane, and vertical motions are

neglected. As this is also the case for the two-body system of interest, where only horizontal motions are of concern, the equation in Z, the vertical force equation, can be dropped. Further, choice of the axis system parallel to the plane of symmetry of the ships eliminates the cross terms of inertia, and, if it is assumed that the center of gravity of each ship is in its centerline plane, (as, indeed, can be expected), the quantity y_g is zero for both ships. The usual assumption that rolling and pitching motions, if not extreme, have little influence on steering and maneuvering is also made, allowing the equations for K and M to be neglected, and making the variables p and q zero. (See Table II-1 for definitions). The equations of motion for a single surface ship moving in the horizontal plane thus become:

$$\text{a)} \quad X = m[\dot{u} - rv - x_g r^2]$$
$$\text{b)} \quad Y = m[\dot{v} + ru + x_g \dot{r}]$$
$$\text{c)} \quad N = I_z \dot{r} + m x_g (\dot{v} + ru) \qquad \text{(2-1)}$$

(where the notation $\dot{v} = \partial v/\partial t$, is used)

The left-hand sides of these equations represent the sum of all forces in the x and y directions and all moments about the z axis, respectively. The right-hand sides are the dynamic response terms associated with the ship and its motions.

IV. Taylor Expansion of Forces and Moments

The forces and moments on the left-hand sides of the equations of motion can be expressed as functions of the properties of the body, properties of the fluid and properties

TABLE II-1

Definitions of Symbols	
x_o, y_o, z_o	Reference coordinates fixed with respect to earth.
x, y, z	Reference coordinates fixed in ship.
u, v, w	Velocities in direction of x, y and z axes, respectively.
ϕ, θ, ψ	Roll, pitch and yaw angles about x, y and z axes, respectively.
p, q, r	Angular velocities about x, y and z axes, respectively. ($p=\dot{\phi}$, $q=\dot{\theta}$, $r=\dot{\psi}$).
δ	Rudder deflection in radians (port rudder is positive).
n	Propeller speed in revolutions per second.
X, Y, Z	Sums of forces acting on ship in x, y, z directions, respectively.
K, M, N	Sums of moments acting on ship about x, y, z axes, respectively.
α	Longitudinal separation of ships measured from Ship A midships section to Ship B midships section.
β	Lateral separation of ships measured from Ship A centerline to Ship B centerline.
Y_{u_B}	Typical of notation used for hydrodynamic derivatives. In this case: $\partial Y/\partial u$ for Ship B, measured when all variables but u are at equilibrium values.
Δt_1, Δt_2	Control system time lags associated with rudder and propeller response, respectively. (Subscripts A or B indicate applicable ship, as Δt_{1_A}).

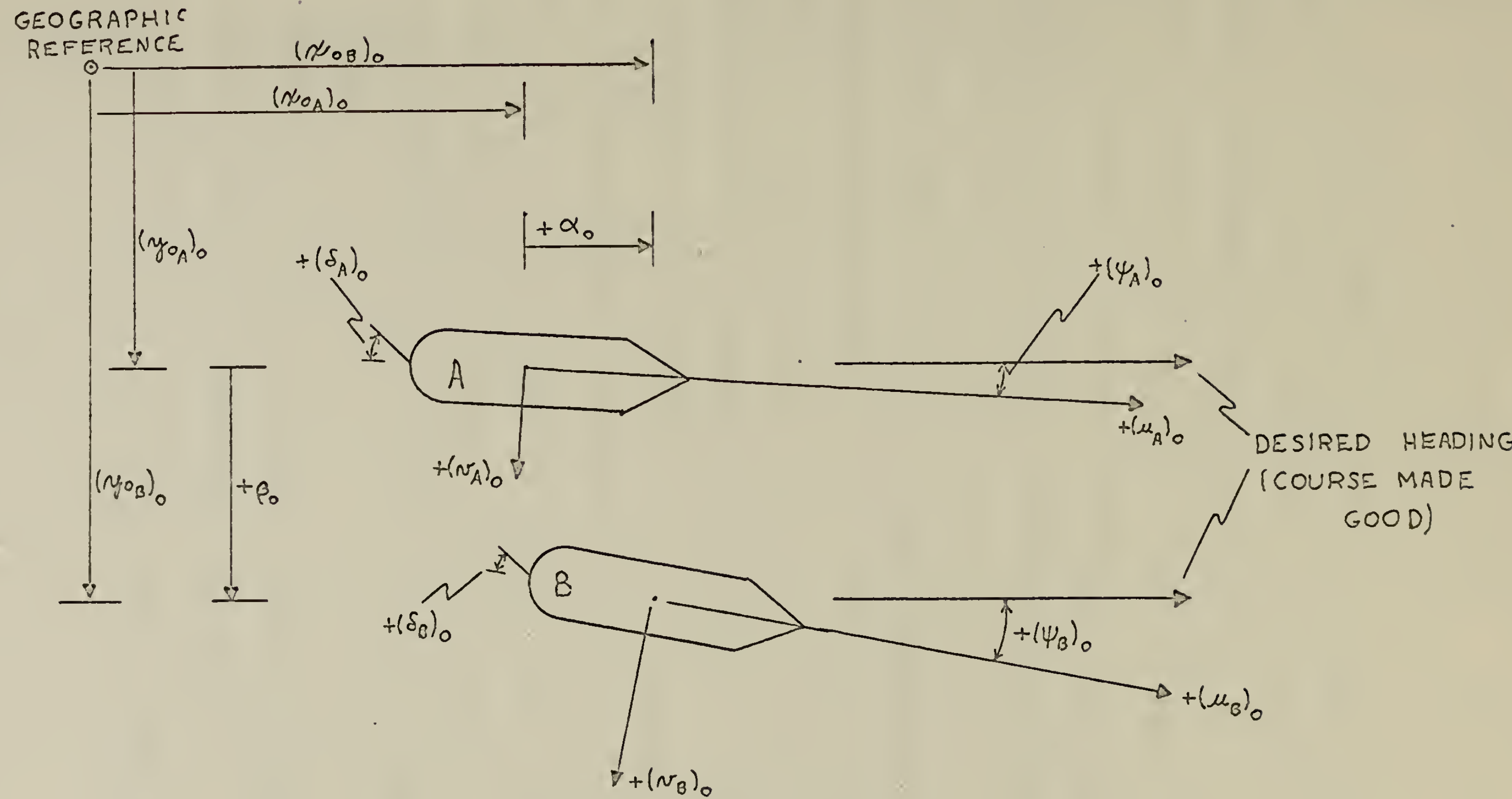
k_{i_A}, k_{i_B}	Control system constants, i=1 to 6, for Ship A or B. Constants associated with control system response to deviations from equilibrium, as $\delta_A = k_{1_A} \Delta\psi_A$.
x_g, y_g, z_g	Distance of center of gravity of ship, (G), from the axis origin, (O), in x, y, z directions respectively.
m_A, m_B	Mass of Ship A or B in units of (lb sec^2/ft)
$\mathcal{D}$	The operator $\partial/\partial t$, as $\mathcal{D} u = \partial u/\partial t$

of the motion. When considering specific hull forms, and using generally accepted scaling laws, the forces and moments may be considered functions of only the motion and orientation parameters of the bodies. (When dealing with automatic control of rudder angle and propeller speed, they are also functions of rudder deflection, δ, and propeller speed, n.)

The reference system for the two-ship system at equilibrium and an illustration of the variables involved are shown in Figure II-2.

When considering motion in an unrestricted horizontal plane, it is clear that no forces or moments arise from a change in ship location with respect to a fixed geographical reference point, allowing x_o and y_o to be discarded as variables of interest. Similarly, the equilibrium yaw angles, ($(\psi_A)_o$ and $(\psi_B)_o$), or drift angles, can be dropped from the list of independent variables because forces and moments arising from a drift angle are accounted for by the terms in u and v. (If there were no drift angle, u would coincide with velocity made good in the desired direction, and v would be zero. Thus the forces and moments that $(\psi_A)_o$ and $(\psi_B)_o$ cause to be introduced are taken up in the terms in u and v. The inter-relation of drift angle and the independent variables, u and v, can be seen from tan(drift angle) = v/u).

It is also possible to drop $\dot{\alpha}$, $\ddot{\alpha}$, $\dot{\beta}$ and $\ddot{\beta}$ as independent variables because of the simple trigonometric relations between them and v, $\dot{v}$, u and $\dot{u}$. The usual assumption that forces and moments produced by $\dot{\delta}$, $\ddot{\delta}$ and $\dot{n}$ are negligible, in



Reference System for Two Ship System

FIGURE II-2

terms of ship control, is also made. (These forces and moments are not necessarily negligible when considering forces on the rudder stock, propeller blades, etc. But they do have a negligible effect on ship motion. (See reference A-4, pg. 7).)

As Abkowitz has shown, the left-hand sides of the equations of motion, comprising the functions describing the forces and moments acting on the ship, can be developed into useful form for analysis by use of the Taylor Series expansion for a function of several variables. The force or moment caused by a change in any variable is expressed as the product of the derivative of the force with respect to that variable, (with all other variables at equilibrium values), and the change in the variable. In the linearized form of the equations, only the linear terms are retained, though the expressions can be written to any desired degree of accuracy, by retaining sufficient terms in the expansion. (For example, the Y force caused by a change in u is written as $(\partial Y/\partial u) \times (\Delta u)$ where $(\partial Y/\partial u)$ is the value of $(\partial Y/\partial u)$ with all other variables at their equilibrium values. The derivatives, such as $(\partial Y/\partial u)$ are the hydrodynamic derivatives and can be experimentally determined.)

Employing standard notation, where Y_u represents $(\partial Y/\partial u)$ at equilibrium, the Y equations, for example, for the two ships, retaining only the linear terms of the Taylor expansion, become:

Ship A:

a) $Y_{\alpha_A} \Delta\alpha + Y_{\beta_A} \Delta\beta + Y_{u_A} \Delta u_A + Y_{\dot{u}_A} \Delta\dot{u}_A + Y_{v_A} \Delta v_A + \ldots$

$\ldots + Y_{\delta_A} \Delta\delta_A + Y_{n_A} \Delta n_A = m_A[\dot{v}_A + r_A u_A + x_{g_A} \dot{r}_A]$

b) $Y_{\alpha_B} \Delta\alpha + Y_{\beta_B} \Delta\beta + Y_{u_B} \Delta u_B + Y_{\dot{u}_B} \Delta\dot{u}_B + Y_{v_B} \Delta v_B + \ldots$

$$\ldots + Y_{\delta_B} \Delta\delta_B + Y_{n_B} \Delta n_B = m_B[\dot{v}_B + r_B u_B + x_{g_B} \dot{r}_B] \qquad (2\text{-}2)$$

The left-hand sides of the X and N equations are similarly expanded for both ships.

V. Linearizing the Dynamic Response Terms

Since only linear terms have been retained in the expanded force and moment expressions on the left-hand sides of the equations, a similar linearization of the dynamic response terms on the right-hand sides must be made.

For all variables, the following holds true:

$$\text{variable} = (\text{variable})_o + \Delta\text{variable} \qquad (2\text{-}3)$$

where the subscript "o" refers to the equilibrium value.

For all time derivatives of the variables, ($\dot{v}$, $\dot{\psi}$, etc.), the equilibrium value is zero, as the equilibrium condition is unchanging in time. Thus for these:

$$\text{variable} = (\text{variable})_o + \Delta\text{variable} = \Delta\text{variable} \qquad (2\text{-}4)$$

(e.g. $\dot{u} = (\dot{u})_o + \Delta\dot{u} = \Delta\dot{u}$)

Making use of (2-3) and (2-4), and by neglecting second

order terms consisting of products of small quantities, such as $(\Delta r)\cdot(\Delta u)$, the linearized right-hand sides of the Y equations become:

$$Y = m[\dot{v} + u_o r + x_g \dot{r}] \qquad (2\text{-}5)$$

Similar developments are applicable to the X and N equations and are carried out in detail in [A-3].

VI. Linearized Equations of Motion

Taking into account all developments outlined thus far, the linearized equations of motion become:

Ship A:

a) $X_{\alpha_A} \Delta\alpha + X_{\beta_A} \Delta\beta + X_{u_A} \Delta u_A + X_{\dot{u}_A} \dot{u}_A + X_{v_A} \Delta v_A + X_{\dot{v}_A} \dot{v}_A$

$+X_{\dot{\psi}_A} \dot{\psi}_A + X_{\ddot{\psi}_A} \ddot{\psi}_A + X_{\delta_A} \Delta\delta_A + X_{n_A} \Delta n_A = m_A \dot{u}_A$

b) $Y_{\alpha_A} \Delta\alpha + Y_{\beta_A} \Delta\beta + Y_{u_A} \Delta u_A + Y_{\dot{u}_A} \dot{u}_A + Y_{v_A} \Delta v_A + Y_{\dot{v}_A} \dot{v}_A$

$+Y_{\dot{\psi}_A} \dot{\psi}_A + Y_{\ddot{\psi}_A} \ddot{\psi}_A + Y_{\delta_A} \Delta\delta_A + Y_{n_A} \Delta n_A = m_A [\dot{v}_A + u_{o_A} \dot{\psi}_A + x_{g_A} \ddot{\psi}_A]$

c) $N_{\alpha_A} \Delta\alpha + N_{\beta_A} \Delta\beta + N_{u_A} \Delta u_A + N_{\dot{u}_A} \dot{u}_A + N_{v_A} \Delta v_A + N_{\dot{v}_A} \dot{v}_A$

$+N_{\dot{\psi}_A} \dot{\psi}_A + N_{\ddot{\psi}_A} \ddot{\psi}_A + N_{\delta_A} \Delta\delta_A + N_{n_A} \Delta n_A = I_{z_A} \ddot{\psi}_A + m_A x_{g_A} (\dot{v}_A + u_{o_A} \dot{\psi}_A)$

d)
e) Corresponding Equations for Ship B
f) (2-6)

VII. Automatic Control

The automatic control system used to maintain each ship in position alongside is assumed capable of deflecting the rudder and changing propeller speed in response to measurement of the ship's heading angle error from equilibrium, $\Delta\psi$, and the separation parameter errors from equilibrium, $\Delta\alpha$ and $\Delta\beta$, as well as the rates of change of these variables. The rudder control is sensitive to ψ, $\dot{\psi}$, β and $\dot{\beta}$; the propeller control to α and $\dot{\alpha}$.

$$\text{a) } \delta = k_1\Delta\psi + k_2\dot{\psi} + k_3\Delta\beta + k_4\dot{\beta}$$

$$\text{b) } n = k_5\Delta\alpha + k_6\dot{\alpha} \qquad (2\text{-}7)$$

(where k_i are the control system constants)

There will be time lags, (Δt), in both control systems, representing the lapse between ordering a control response and attainment of the response. Thus rudder deflection at time t is proportional to $\dot{\psi}$ at time t_1, (where $t = t_1+\Delta t$). Therefore:

$$\delta(t) = k_1\psi(t-\Delta t) + k_2\dot{\psi}(t-\Delta t) + k_3\beta(t-\Delta t) + k_4\dot{\beta}(t-\Delta t) \qquad (2\text{-}8)$$

In the linearized equations of motion ψ and β are functions of t. Thus these variables in (2-8) must be obtained as functions of t, which is done through another Taylor expansion, carried out in detail in [A-3]. If the time lags are assumed small, only linear terms need be kept, and the automatic control system equations for δ and n become:

a) $\delta = [k_1+(k_2-k_1\Delta t_1)\mathcal{D} - k_2\Delta t_1\mathcal{D}^2]\Delta\psi$

$+[k_3+(k_4-k_3\Delta t_1)\mathcal{D} - k_4\Delta t_1\mathcal{D}^2]\Delta\beta$

b) $n = [k_5+(k_6-k_5\Delta t_2)\mathcal{D} - k_6\Delta t_2\mathcal{D}^2]\Delta\alpha$ (2-9)

(Where $\mathcal{D}$ is the operator $\partial/\partial t$; e.g. $\mathcal{D}^2\psi = \partial^2\psi/\partial t^2$)

Here Δt_1 is the time lag associated with rudder response and Δt_2 that associated with propeller response.

These expressions for δ and n are substituted in the equations of motion.

VIII. Equations Relating α and β to u, v and ψ

Examination of Figure II-2 shows the following relationships to hold:

a) $\dot{\alpha} = u_B\cos\psi_B - u_A\cos\psi_A + v_A\sin\psi_A - v_B\sin\psi_B$

b) $\dot{\beta} = u_B\sin\psi_B - u_A\sin\psi_A + v_B\cos\psi_B - v_A\cos\psi_A$ (2-10)

Since ψ_A and ψ_B are small, $\cos\psi_A \approx 1.0$ and $\cos\psi_B \approx 1.0$. And since $v_A \ll u_A$ and $v_B \ll u_B$, and $\sin\psi_A$ and $\sin\psi_B$ are small, the products $v_A\sin\psi_A$ and $v_B\sin\psi_B$ are second order. Thus (2-10), a), becomes:

$\dot{\alpha} \approx u_B - u_A = ((u_B)_0 + \Delta u_B) - ((u_A)_0 + \Delta u_A)$

where $(u_B)_0 = u_0 = (u_A)_0$, and:

$\dot{\alpha} \approx \Delta u_B - \Delta u_A$

Δ tan x considered ≈ 0

Also, since ψ_A and ψ_B are small, $\sin\psi_A \approx \psi_A$ and $\sin\psi_B \approx \psi_B$. Thus, (2-10), b), becomes:

$$\dot{\beta} \approx u_B\psi_B - u_A\psi_A + \Delta v_B - \Delta v_A$$

$$\approx \psi_B(u_B)_0 - \psi_A(u_A)_o + \underbrace{\psi_B\Delta u_B - \psi_A\Delta u_A}_{\text{2nd order}} + \Delta v_B - \Delta v_A$$

Therefore, the equations, (2-10), become:

a) $\dot{\alpha} \approx \Delta u_B - \Delta u_A$

b) $\dot{\beta} \approx \underbrace{u_o\Delta\psi_B - u_o\Delta\psi_A}_{\approx 0} + \Delta v_B - \Delta v_A$ (2-11)

IX. The Complete Set of Linear Equations of Motion for the Two Ship System

Incorporating all the foregoing discussion, the linear equations of motion for the two-ship system emerge:

Ship A: (2-12)

a) $X_{\alpha_A}\Delta\alpha + X_{\beta_A}\Delta\beta + X_{u_A}\Delta u_A + X_{\dot{u}_A}\dot{u}_A + X_{v_A}\Delta v_A + X_{\dot{v}_A}\dot{v}_A + X_{\dot{\psi}_A}\dot{\psi}_A$

$+X_{\ddot{\psi}_A}\ddot{\psi}_A + X_{\delta_A}[k_{1_A} + (k_{2_A} - k_{1_A}\Delta t_{1_A})\mathcal{D} - k_{2_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\psi_A$

$+X_{\delta_A}[k_{3_A} + (k_{4_A} - k_{3_A}\Delta t_{1_A})\mathcal{D} - k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$

$+X_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D} - k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha - m_A\dot{u}_A = 0$

$$\text{b)}\ Y_{\alpha_A}\Delta\alpha + Y_{\beta_A}\Delta\beta + Y_{u_A}\Delta u_A + Y_{\dot{u}_A}\dot{u}_A + Y_{v_A}\Delta v_A + Y_{\dot{v}_A}\dot{v}_A + Y_{\dot{\psi}_A}\dot{\psi}_A$$

$$+Y_{\ddot{\psi}_A}\ddot{\psi}_A + Y_{\delta_A}[k_{1_A} + (k_{2_A} - k_{1_A}\Delta t_{1_A})\mathcal{D} - k_{2_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\psi_A$$

$$+Y_{\delta_A}[k_{3_A} + (k_{4_A} - k_{3_A}\Delta t_{1_A})\mathcal{D} - k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$$

$$+Y_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D} - k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$-m_A[\dot{v}_A + u_o\dot{\psi}_A + x_{g_A}\ddot{\psi}_A] = 0$$

$$\text{c)}\ N_{\alpha_A}\Delta\alpha + N_{\beta_A}\Delta\beta + N_{u_A}\Delta u_A + N_{\dot{u}_A}\dot{u}_A + N_{v_A}\Delta v_A + N_{\dot{v}_A}\dot{v}_A + N_{\dot{\psi}_A}\dot{\psi}_A$$

$$+N_{\ddot{\psi}_A}\ddot{\psi}_A + N_{\delta_A}[k_{1_A} + (k_{2_A} - k_{1_A}\Delta t_{1_A})\mathcal{D} - k_{2_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\psi_A$$

$$+N_{\delta_A}[k_{3_A} + (k_{4_A} - k_{3_A}\Delta t_{1_A})\mathcal{D} - k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$$

$$+N_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D} - k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$-I_{z_A}\ddot{\psi}_A - m_A x_{g_A}(\dot{v}_A + u_o\dot{\psi}_A) = 0$$

d)
e) Precisely Similar Equations for Ship B.
f)

g) $\dot{\alpha} = \Delta u_B - \Delta u_A = \mathcal{D}\alpha$

h) $\dot{\beta} = u_o\Delta\psi_B - u_o\Delta\psi_A + \Delta v_B - \Delta v_A = \mathcal{D}\beta$

X. Identifying Zero-Value Derivatives

Arguing from symmetry of the ship about its centerline plane, Abkowitz demonstrates that X_v, the change in x-direction forces of the ship caused by a change in v, with all other variables fixed at their equilibrium values, must have a value of zero. Similarly, it is demonstrated that $X_{\dot{v}} = X_{\dot{\psi}} = X_{\ddot{\psi}} = 0$. ([A-3], pp. I-28 and I-30). Due to the port and starboard symmetry of the rudder, X_δ is also zero. ([A-3], pg. I-67).

Further arguments from symmetry demonstrate that in the open water case, Y_u, $Y_{\dot{u}}$, N_u and $N_{\dot{u}}$ are all zero. But for the two-ship system of interest, the interaction side force and yaw moment are affected by changes in speed. Thus Y_u and N_u cannot be assumed to be zero. The effect on this force and moment, of a small acceleration, is, however, still assumed negligible, and $Y_{\dot{u}}$ and $N_{\dot{u}}$ are assumed to be zero.

On examining the hydrodynamic derivatives due to the orientation of the ships with respect to each other, further identification of negligible derivatives can be made. The derivatives X_α and X_β, representing, respectively, the change in x-direction force caused by small changes in longitudinal and lateral separations of the ships, will, at most, be small. Experiments have shown that interaction phenomena, while affecting Y forces and N moments, have no measurable effect on the forward motion of the ships, or on their resistance forces. ([A-1], [A-2]) Thus, X_α and X_β are set equal to zero.

Summarizing:

$$X_v = X_{\dot{v}} = X_{\dot{\psi}} = X_{\ddot{\psi}} = X_{\delta} = X_{\alpha} = X_{\beta} = Y_{\dot{u}} = N_{\dot{u}} = 0 \tag{2-13}$$

With this information, the equations of motion are simplified to:

Ship A: (2-14)

a) $$X_{u_A}\Delta u_A + X_{\dot{u}_A}\dot{u}_A + X_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D}$$

$$- k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha - m_A\dot{u}_A = 0$$

b) $$Y_{\alpha_A}\Delta\alpha + Y_{\beta_A}\Delta\beta + Y_{u_A}\Delta u_A + Y_{v_A}\Delta v_A + Y_{\dot{v}_A}\dot{v}_A + Y_{\dot{\psi}_A}\dot{\psi}_A$$

$$+ Y_{\ddot{\psi}_A}\ddot{\psi}_A + Y_{\delta_A}[k_{1_A} + (k_{2_A} - k_{1_A}\Delta t_{1_A})\mathcal{D} - k_{2_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\psi_A$$

$$+ Y_{\delta_A}[k_{3_A} + (k_{4_A} - k_{3_A}\Delta t_{1_A})\mathcal{D} - k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$$

$$+ Y_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D} - k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$- m_A(\dot{v}_A + u_o\dot{\psi}_A + x_{g_A}\ddot{\psi}_A) = 0$$

c) $N_{\alpha_A}\Delta\alpha + N_{\beta_A}\Delta\beta + N_{u_A}\Delta u_A + N_{v_A}\Delta v_A + N_{\dot{v}_A}\dot{v}_A + N_{\dot{\psi}_A}\dot{\psi}_A$

$+N_{\ddot{\psi}_A}\ddot{\psi}_A + N_{\delta_A}[k_{1_A} + (k_{2_A} - k_{1_A}\Delta t_{1_A})\mathcal{D} - k_{2_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\psi_A$

$+N_{\delta_A}[k_{3_A} + (k_{4_A} - k_{3_A}\Delta t_{1_A})\mathcal{D} - k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$

$+N_{n_A}[k_{5_A} + (k_{6_A} - k_{5_A}\Delta t_{2_A})\mathcal{D} - k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$

$-I_{z_A}\ddot{\psi}_A - m_A x_{g_A}(\dot{v}_A + u_o\dot{\psi}_A) = 0$

d)
e) Precisely Similar Equations for Ship B.
f)

g) $\mathcal{D}\alpha = \Delta u_B - \Delta u_A$

h) $\mathcal{D}\beta = u_o\Delta\psi_B - u_o\Delta\psi_A + \Delta v_B - \Delta v_A$

XI. Rearranging Equations of Motion

At this point, terms are gathered so as to have each variable multiplied by the sum of all its coeffieients in each equation.

Remembering that $\mathcal{D}$(variable) = ∂(variable)/∂t, then:

$$\mathcal{D}(\Delta u) = \frac{\partial(\Delta u)}{\partial t} = \frac{\partial(u-u_o)}{\partial t} = \frac{\partial u}{\partial t} - \frac{\partial u_o}{\partial t} \qquad (2\text{-}15)$$

But u_o is constant, so $\mathcal{D}(\Delta u) = \mathcal{D}u$.

Thus:

Ship A: (2-16)

a) $$[X_{n_A}k_{5_A}+X_{n_A}(k_{6_A}-k_{5_A}\Delta t_{2_A})\mathcal{D}-X_{n_A}k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$+[X_{u_A}+(X_{\dot{u}_A}-m_A)\mathcal{D}]\Delta u_A = 0$$

b) $$[(Y_{\alpha_A}+Y_{n_A}k_{5_A})+Y_{n_A}(k_{6_A}-k_{5_A}\Delta t_{2_A})\mathcal{D}-Y_{n_A}k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$+[(Y_{\beta_A}+Y_{\delta_A}k_{3_A})+Y_{\delta_A}(k_{4_A}-k_{3_A}\Delta t_{1_A})\mathcal{D}-Y_{n_A}k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$$

$$+[Y_{u_A}]\Delta u_A+[Y_{v_A}+(Y_{\dot{v}_A}-m_A)\mathcal{D}]\Delta v_A$$

$$+[Y_{\delta_A}k_{1_A}+((Y_{\dot{\psi}_A}-m_Au_o)+Y_{\delta_A}(k_{2_A}-k_{1_A}\Delta t_{1_A}))\mathcal{D}$$

$$+((Y_{\ddot{\psi}_A}-m_Ax_{g_A})-Y_{\delta_A}k_{2_A}\Delta t_{1_A})\mathcal{D}^2]\Delta\psi_A = 0$$

c) $$[(N_{\alpha_A}+N_{n_A}k_{5_A})+N_{n_A}(k_{6_A}-k_{5_A}\Delta t_{2_A})\mathcal{D}-N_{n_A}k_{6_A}\Delta t_{2_A}\mathcal{D}^2]\Delta\alpha$$

$$+[(N_{\beta_A}+N_{\delta_A}k_{3_A})+N_{\delta_A}(k_{4_A}-k_{3_A}\Delta t_{1_A})\mathcal{D}-N_{\delta_A}k_{4_A}\Delta t_{1_A}\mathcal{D}^2]\Delta\beta$$

$$+N_{u_A}\Delta u_A+[N_{v_A}+(N_{\dot{v}_A}-m_Ax_{g_A})\mathcal{D}]\Delta v_A+[N_{\delta_A}k_{1_A}+((N_{\dot{\psi}_A}-m_Ax_{g_A}u_o)$$

$$+N_{\delta_A}(k_{2_A}-k_{1_A}\Delta t_{1_A}))\mathcal{D}+((N_{\ddot{\psi}_A}-I_{z_A})-N_{\delta_A}k_{2_A}\Delta t_{1_A})\mathcal{D}^2]\Delta\psi_A = 0$$

In writing the equations for Ship B use is made of the relationships provided by equation g and h of (2-14), to substitute for Δu_B and Δv_B in the Ship B equations. From these equations, $\Delta u_B = \Delta u_A + \mathcal{D}\alpha$ and

$\Delta v_B = \mathcal{D}\beta + \Delta v_A + u_o\Delta\psi_A - u_o\Delta\psi_B$. Thus:

Ship B: (2-16)

$$d)\ [X_{n_B}k_{5_B} + (X_{n_B}(k_{6_B} - k_{5_B}\Delta t_{2_B}) + X_{u_B})\mathcal{D}$$

$$+((X_{\dot{u}_B} - m_B) - X_{n_B}k_{6_B}\Delta t_{2_B})\mathcal{D}^2]\Delta\alpha$$

$$+[X_{u_B} + (X_{\dot{u}_B} - m_B)\mathcal{D}]\Delta u_A = 0$$

$$e)\ [(Y_{\alpha_B} + Y_{n_B}k_{5_B}) + (Y_{n_B}(k_{6_B} - k_{5_B}\Delta t_{2_B}) + Y_{u_B})\mathcal{D} - Y_{n_B}k_{6_B}\Delta t_{2_B}\mathcal{D}^2]\Delta\alpha$$

$$+[(Y_{\beta_B} + Y_{\delta_B}k_{3_B}) + (Y_{\delta_B}(k_{4_B} - k_{3_B}\Delta t_{1_B}) + Y_{v_B})\mathcal{D}$$

$$+((Y_{\dot{v}_B} - m_B) - Y_{\delta_B}k_{4_B}\Delta t_{1_B})\mathcal{D}^2]\Delta\beta + [Y_{u_B}]\Delta u_A + [Y_{v_B} + (Y_{\dot{v}_B} - m_B)\mathcal{D}]\Delta v_A$$

$$+[u_oY_{v_B} + u_o(Y_{\dot{v}_B} - m_B)\mathcal{D}]\Delta\psi_A$$

$$+\ [(Y_{\delta_B}k_{1_B} - u_oY_{v_B}) + ((Y_{\dot{\psi}_B} - m_Bu_o) + Y_{\delta_B}(k_{2_B} - k_{1_B}\Delta t_{1_B})$$

$$-u_o(Y_{\dot{v}_B} - m_B))\mathcal{D} + ((Y_{\ddot{\psi}_B} - m_Bx_{g_B}) - Y_{\delta_B}k_{2_B}\Delta t_{1_B})\mathcal{D}^2]\Delta\psi_B = 0$$

f) $[(N_{\alpha_B}+N_{n_B}k_{5_B})+(N_{n_B}(k_{6_B}-k_{5_B}\Delta t_{2_B})+N_{u_B})\mathcal{D}-N_{n_B}k_{6_B}\Delta t_{2_B}\mathcal{D}^2]\Delta\alpha$

$+[(N_{\beta_B}+N_{\delta_B}k_{3_B})+(N_{\delta_B}(k_{4_B}-k_{3_B}\Delta t_{1_B})+N_{v_B})\mathcal{D}+((N_{\dot{v}_B}-m_Bx_{g_B})$

$-N_{\delta_B}k_{4_B}\Delta t_{1_B}))\mathcal{D}^2]\Delta\beta+[N_{u_B}]\Delta u_A+[N_{v_B}+(N_{\dot{v}_B}-m_Bx_{g_B})\mathcal{D}]\Delta v_A$

$+[u_oN_{v_B}+u_o(N_{\dot{v}_B}-m_Bx_{g_B})\mathcal{D}]\Delta\psi_A+[(N_{\delta_B}k_{1_B}-u_oN_{v_B})$

$+((N_{\dot{\psi}_B}-m_Bx_{g_B}u_o)+N_{\delta_B}(k_{2_B}-k_{1_B}\Delta t_{1_B})-u_o(N_{\dot{v}_B}-m_Bx_{g_B}))\mathcal{D}$

$+((N_{\dot{\psi}_B}-I_{z_B})-N_{\delta_B}k_{2_B}\Delta t_{1_B})\mathcal{D}^2]\Delta\psi_B = 0$

These, then, are the six equations of motion in six variables, in their final form for solution. The equations are of the form: (2-17)

$(AX1)\Delta\alpha + 0 + (AX3)\Delta u_A + 0 + 0 + 0 = 0$

$(AY1)\Delta\alpha + (AY2)\Delta\beta + (AY3)\Delta u_A + (AY4)\Delta v_A + (AY5)\Delta\psi_A + 0 = 0$

$(AN1)\Delta\alpha + (AN2)\Delta\beta + (AN3)\Delta u_A + (AN4)\Delta v_A + (AN5)\Delta\psi_A + 0 = 0$

$(BX1)\Delta\alpha + 0 + (BX3)\Delta u_A + 0 + 0 + 0 = 0$

$(BY1)\Delta\alpha + (BY2)\Delta\beta + (BY3)\Delta u_A + (BY4)\Delta v_A + (BY5)\Delta\psi_A + (BY6)\Delta\psi_B = 0$

$(BN1)\Delta\alpha + (BN2)\Delta\beta + (BN3)\Delta u_A + (BN4)\Delta v_A + (BN5)\Delta\psi_A + (BN6)\Delta\psi_B = 0$

This is a set of six simultaneous, linear differential equations in six unknowns. The term BX3, for instance, is used to denote: Ship B, X-equation, 3rd term, (or Δu_A term). Each of the coefficients is, itself, a polynomial in $\mathcal{D}$, as

can be seen from equations (2-16), for which (2-17) is a shorthand notation.

XII. Solving the Equations of Motion

If the terms AX1, BX1, etc., were just numerical coefficients, straight-forward algebraic techniques could be used to solve this system of equations. Yet, while these coefficients are, themselves, polynominals in the operator $\mathcal{D}$, Abkowitz shows that $\mathcal{D}$ can be treated as an algebraic quantity. ([A-3], pp. I-26, 27). This is possible because the hydrodynamic derivatives are defined as the slope of a force or moment versus a dynamic variable taken at the equilibrium condition. Thus the terms in the coefficients AX1, etc., other than $\mathcal{D}$, are constants in time.

The solution for any of the variables is, then, the determinant of the coefficients, with the column corresponding to the variable being solved for replaced by the right-hand side column vector, divided by the determinant of all the coefficients.

For example:

(see next page)

$$\Delta\alpha = \frac{\begin{vmatrix} 0 & 0 & AX3 & 0 & 0 & 0 \\ 0 & AY2 & AY3 & AY4 & AY5 & 0 \\ 0 & AN2 & AN3 & AN4 & AN5 & 0 \\ 0 & 0 & BX3 & 0 & 0 & 0 \\ 0 & BY2 & BY3 & BY4 & BY5 & BY6 \\ 0 & BN2 & BN3 & BN4 & BN5 & BN6 \end{vmatrix}}{\begin{vmatrix} AX1 & 0 & AX3 & 0 & 0 & 0 \\ AY1 & AY2 & AY3 & AY4 & AY5 & 0 \\ AN1 & AN2 & AN3 & AN4 & AN5 & 0 \\ BX1 & 0 & BX3 & 0 & 0 & 0 \\ BY1 & BY2 & BY3 & BY4 & BY5 & BY6 \\ BN1 & BN2 & BN3 & BN4 & BN5 & BN6 \end{vmatrix}} = \frac{0}{|DET|} \tag{2-18}$$

Evaluation of DET leads to a polynominal in $\mathcal{D}$, (made up of sums of products of polynominals in $\mathcal{D}$, such as AX1, etc.), whose coeffieients are various combination of the hydrodynamic derivatives and control system constants. Thus the solution for each variable would be:

$$\text{variable} = \frac{0}{a_1\mathcal{D}^n + a_2\mathcal{D}^{n-1} + \cdots\cdots + a_{n+1}} \tag{2-19}$$

If DET is other than zero, the solution for all variables would be identically zero for all time after a small disturbance from equilibrium---a physical impossibility. Thus, the denominator, the determinant DET, must be set equal to zero. Setting this polynominal in $\mathcal{D}$, resulting from the evaluation

of DET, equal to zero, the roots of the polynomial can be obtained, where $\sigma_1, \sigma_2, \ldots . \sigma_n$ are these roots. Then:

$$\text{variable} = \frac{0}{(\mathcal{D}-\sigma_n)(\mathcal{D}-\sigma_{n-1})\ldots\ldots(\mathcal{D}-\sigma_1)} \tag{2-20}$$

Abkowitz demonstrates how the operation $\frac{1}{(\mathcal{D}-\sigma)}[z]$ is equivalent to:

$$\frac{1}{(\mathcal{D}-\sigma)}[z] = e^{\sigma t}\int e^{-\sigma t}[z]dt \tag{2-21}$$

Thus, $\frac{0}{(\mathcal{D}-\sigma)} = e^{\sigma t}\int e^{-\sigma t}(0)dt = c_1 e^{\sigma t}$

Carrying out this operation successively to evaluate equation (2-20) yields:

$$\text{variable} = c_1 e^{\sigma_1 t} + c_2 e^{\sigma_2 t} \ldots\ldots\ldots c_n e^{\sigma_n t}$$

where the c_i are arbitrary constants of integration.

XIII. Establishing the Stability Criterion for the System

Since each of the variables is a sum of exponential terms in time, it is clear that the only way the variables can all go to zero as time increases, is for all the σ_i (or all real parts where the σ_i are complex) to be negative. The criterion of stability of the two ship system is for all the variables, representing, as they do, deviations from the equilibrium condition, to go to zero as time increases. For when all the variables have gone to zero, the system has regained the equilibrium condition, meeting the definition of a stable system.

Thus, the stability of the system requires that the real parts of all σ_i be negative. As the σ_i are the roots of the characteristic equation of stability resulting from setting the polynomial in $\mathscr{D}$, (which in turn results from evaluating DET), equal to zero, the stability requirement for the two-ship system is that the real parts of all the roots of DET be less than zero.

Chapter III

Procedure

I. Background

As stated at the conclusion of Chapter I, a modified OLNA/MARINER class merchant hull is the ship used in the mathematical model. As indicated there, this was done because open water hydrodynamic derivatives, in non-dimensional form, were available for a MARINER, ([A-4], [A-5]), and because the only good, quantitative interaction phenomena data available were for a ship quite similar to a MARINER---the R.F.A. OLNA.

II. Open Water Hydrodynamic Derivatives

Reference A-5 presents experimentally determined values for most open water derivatives, measured with a planar motion mechanism, for a MARINER model. Reference A-4 utilized these values, with the exception of X_u, for which a different value was determined. The accuracy of the values used in [A-4] was borne out with full scale sea trials and these values are used in this thesis for the open water derivatives, for ship A, and are listed in Table III-1.

III. Obtaining the Interaction Hydrodynamic Derivatives

A. Y_α, Y_β, N_α, N_β.

Values for hydrodynamic derivatives, as such, arising from interaction effects, could not be obtained. The interaction data for OLNA consisted of dimensionless forces and moments at side-to-side ship separations of 50 and 100 feet,

for all longitudinal positions of the midships sections of the ships between -600 and +600 feet. Also available were force and moment data for OLNA at side-to-side ship separations of from 30 to 150 feet, with the ships abreast. [A-1] (See Figures I-3, I-4 and I-5)

Using the data illustrated by Figures I-3, 4 and 5, the only positions at which Y_α, Y_β, N_α and N_β could all be determined were at side-to-side distances of 50 and 100 feet, with the ships abreast. (Points 1 and 6, Figure III-1). This limitation of the examination of system stability to only two ship positions was felt to be too restrictive. As the only means available for obtaining the desired derivatives at other positions of interest was extrapolation and interpolation of the OLNA data, this course was followed for both Y force and N moment. As data was available at side-to-side distances (STSD, for short), of 50 and 100 feet, these values were chosen as bounds to the range of interest, just as values of -600 and +600 feet were chosen as bounds in the longitudinal direction. The way in which Y force was interpolated in the range of interest will be illustrated.

The Y force was known along three lines of position, as shown in Figures I-3, and I-5, and as in Figure III-1. To determine Y at positions other than those known in Figure III-1, it was assumed that the ratio of change in Y between 1' and 2' to the change between 1' and 6' was the same as the ratio of the change in Y between 1 and 2 to the change between 1 and 6.

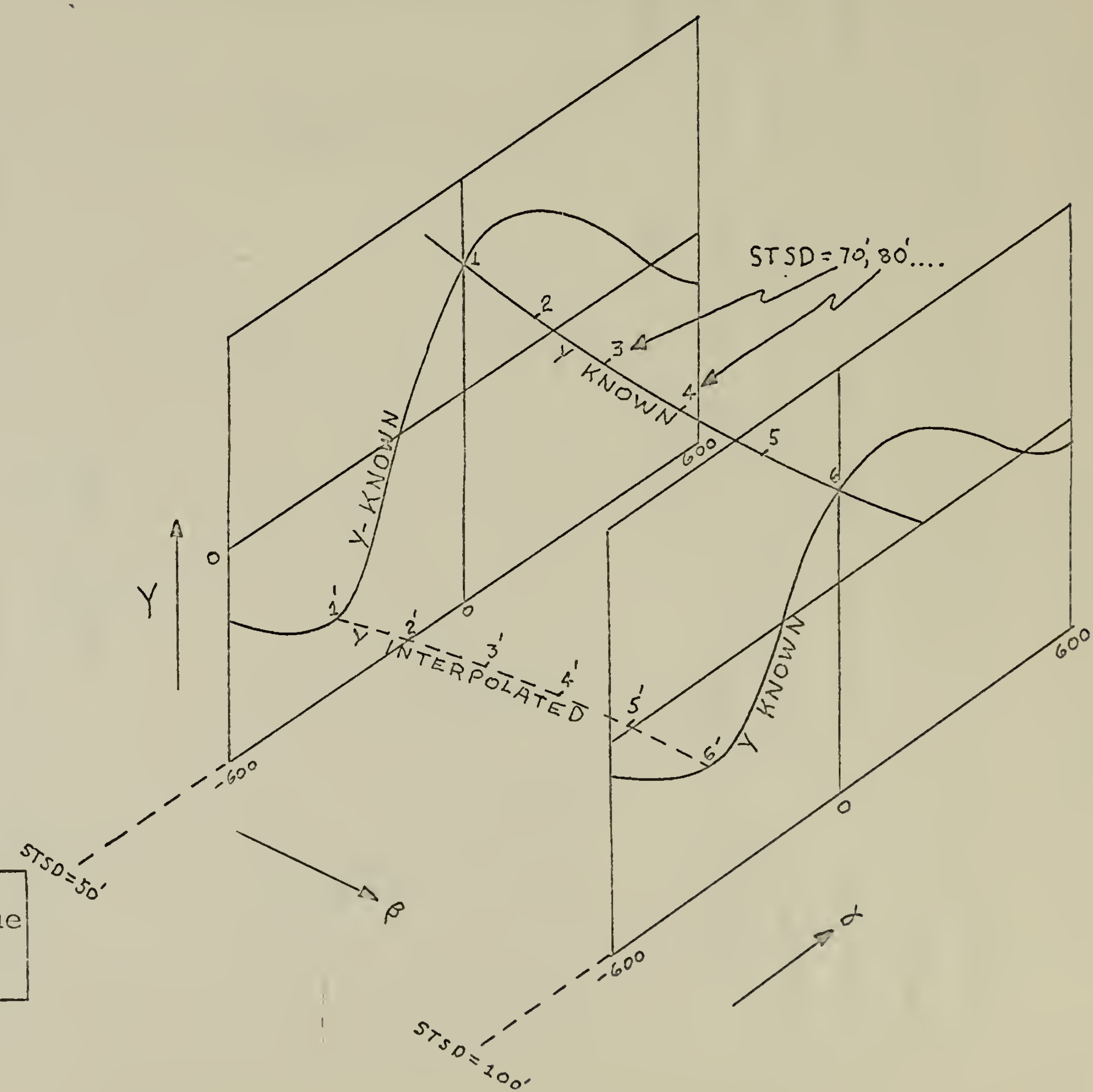


Illustration of Interpolation Technique Used for Y Force and N Moment

Figure III-1

Or:

$$\frac{(Y_{2'}-Y_{1'})}{(Y_{6'}-Y_{1'})} = \frac{(Y_2-Y_1)}{(Y_6-Y_1)} \; ; \; \frac{(Y_{3'}-Y_{1'})}{(Y_{6'}-Y_{1'})} = \frac{(Y_3-Y_1)}{(Y_6-Y_1)} \text{, etc.} \qquad (3\text{-}1)$$

The points 1 through 6 in Figure III-1 were chosen at 10 foot intervals, and the interpolation for these values of STSD=60, 70, 80 and 90 feet was performed at 50 foot intervals in α. Thus, values of Y were obtained at all intersections of a grid with lateral spacing of 10 feet and longitudinal spacing of 50 feet, as shown below:

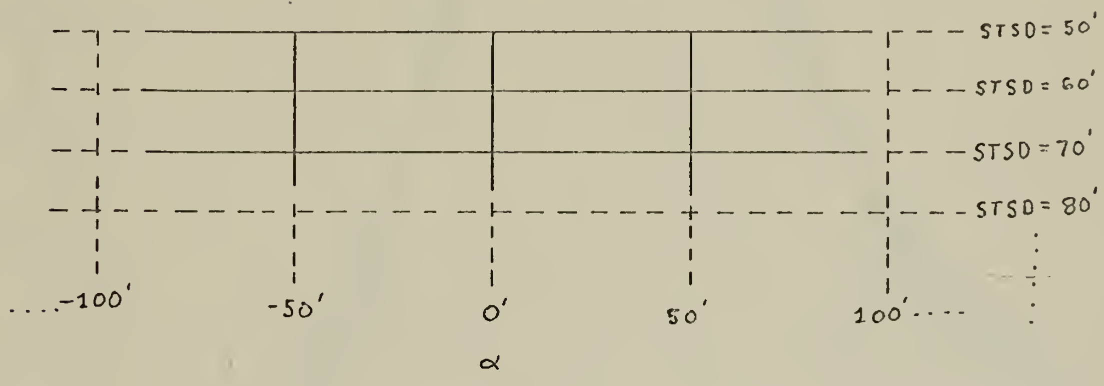
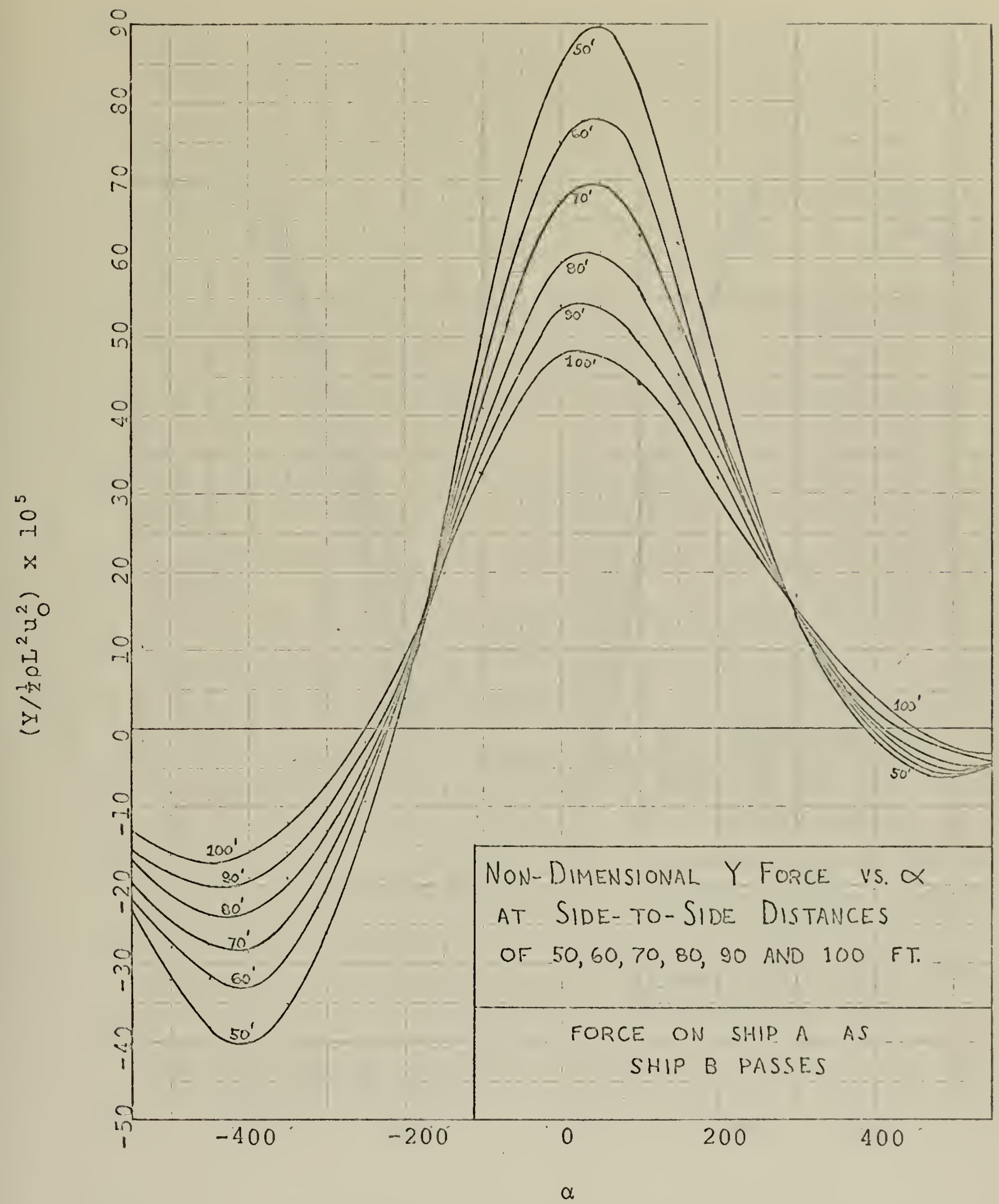


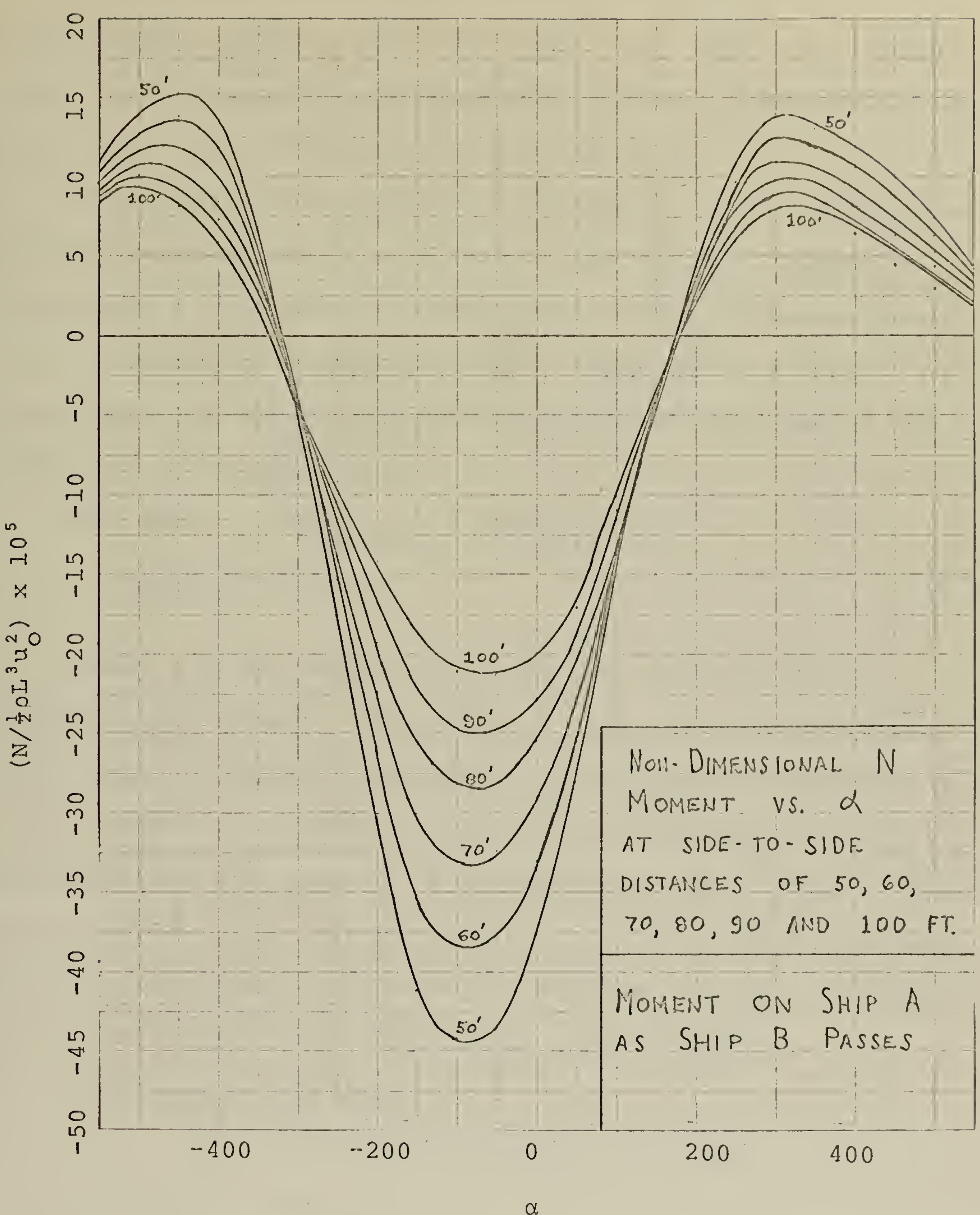
Figure III-2

The results of such interpolations for both Y and N are shown as figures III-3 and III-4, where the curves for STSD = 50 and 100 feet are from [A-1] and the others are interpolated. The interpolated curves are felt to give a reasonable picture of the way the interaction force and moment would be expected to vary as STSD is changed. And, of course, the points on the line α=0 are the same points as those plotted in Figure I-5.



Dimensionless Y Force vs. Longitudinal Separation

<u>FIGURE III-3</u>



Dimensionless N Moment vs. Longitudinal Separation

FIGURE III-4

Obtaining the interaction forces and moments over this range of 50' $\leq$ STSD $\leq$ 100' and -600' $\leq \alpha \leq$ +600' is, though, just a step along the way to obtaining the derivatives Y_α, Y_β, N_α and N_β, which are what is really desired.

The longitudinal separation parameter, α, is represented directly on the curves of Y and N. The lateral separation parameter, β, is related to the side-to-side distances shown on the figures by a constant, for a given pair of ships. Since β is the lateral separation of the centerlines of the ships, it is apparent that:

$$\beta = \text{STSD} + (\text{BEAM}_{\text{Ship A}} + \text{BEAM}_{\text{Ship B}})/2$$

$$= \text{STSD} + \text{Constant} \qquad (3\text{-}2)$$

Thus, for two ships of 50' beam, an STSD of 50' is equivalent to β=100'. Therefore, Figure III-3, for example, represents a surface in space, (as sketched in Figure III-1), whose horizontal coordinates are α and β and whose vertical coordinate is the value of Y everywhere on the surface. Then, to get $\partial Y/\partial\alpha$, or Y_α, at any point, the slope of the Y surface in the α direction, at the point, must be obtained. Similarly, Y_β is the slope in the β direction. For example:

a) $Y(\alpha,\beta) = Y(100,60)$

$$= ((Y(150,60)-Y(50,60))/(150-50)$$

b) $Y(\alpha,\beta) = Y(100,60)$

$$= ((Y(100,70)-Y(100,50))/(70-50) \qquad (3\text{-}3)$$

In this way, Y_α everywhere in the range (50'$\leq$STSD$\leq$100';

$-550' \leq \alpha \leq 550'$), was obtained, as was Y_β everywhere in the range ($60' \leq STSD \leq 90'$; $-600' \leq \alpha \leq 600'$). (The values of Y_α and Y_β at the extremities of the range of known Y values, in the direction of differentiation, could not, of course, be obtained by this simple procedure).

The values of N_α and N_β over the same ranges were obtained in the same way.

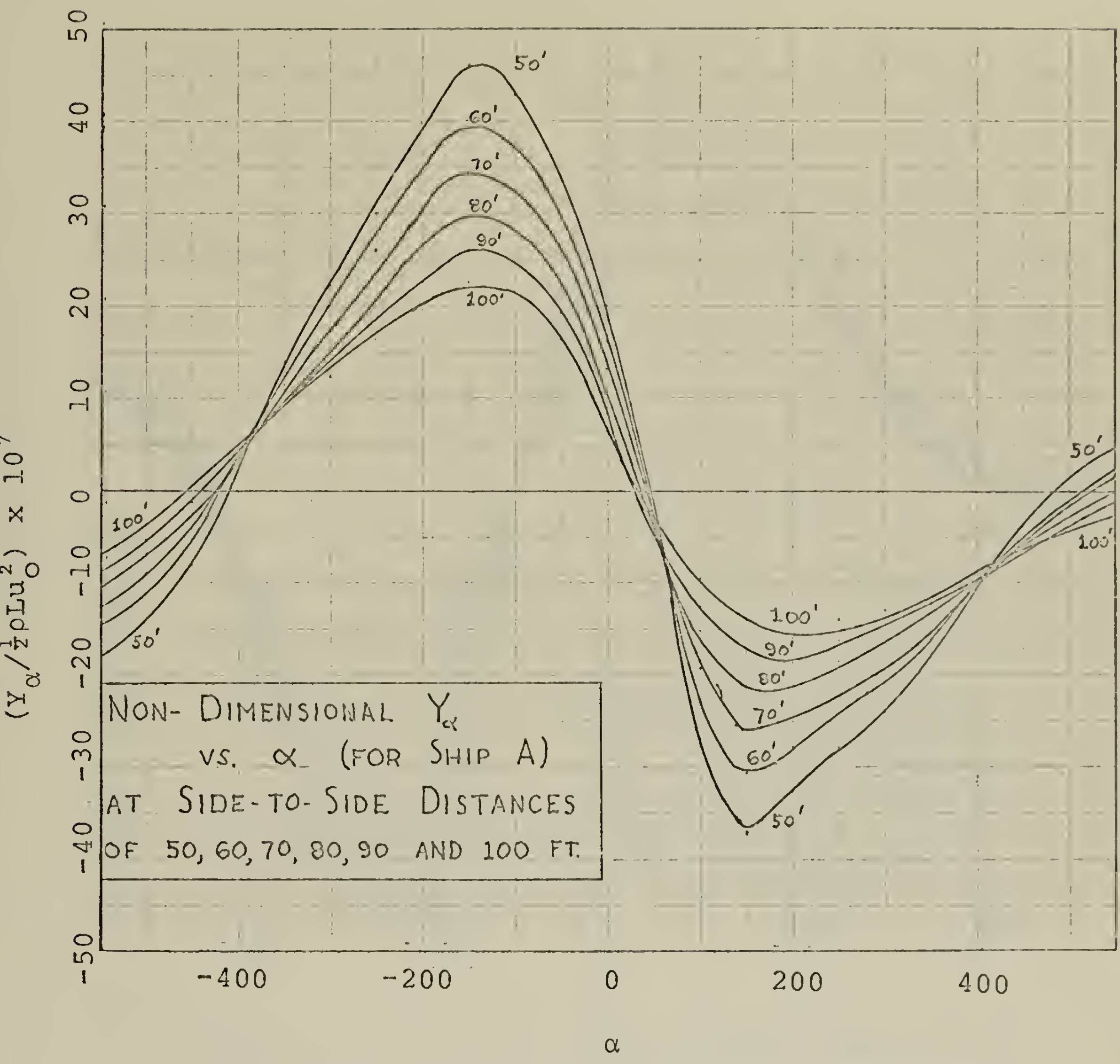
A computer program, program INTCOF, was written to take the values of Y and N at the locations known from Figures I-3, 4 and 5, to interpolate to get Y and N throughout the range, and then determine Y_α, Y_β, N_α and N_β in this range. The curves in Figures III-3 and III-4 for side-to-side distances of 60, 70, 80 and 90 feet are plots of the Y and N outputs of the INTCOF interpolations.

Similarly, the curves in Figures III-5 and III-6 for STSD 60, 70, 80 and 90 feet are INTCOF outputs. And in Figures III-7 and III-8, the curves are all INTCOF outputs.

Program INTCOF is described in detail in Appendix II, and the results of the program for the modified OLNA/MARINER ship are included therein.

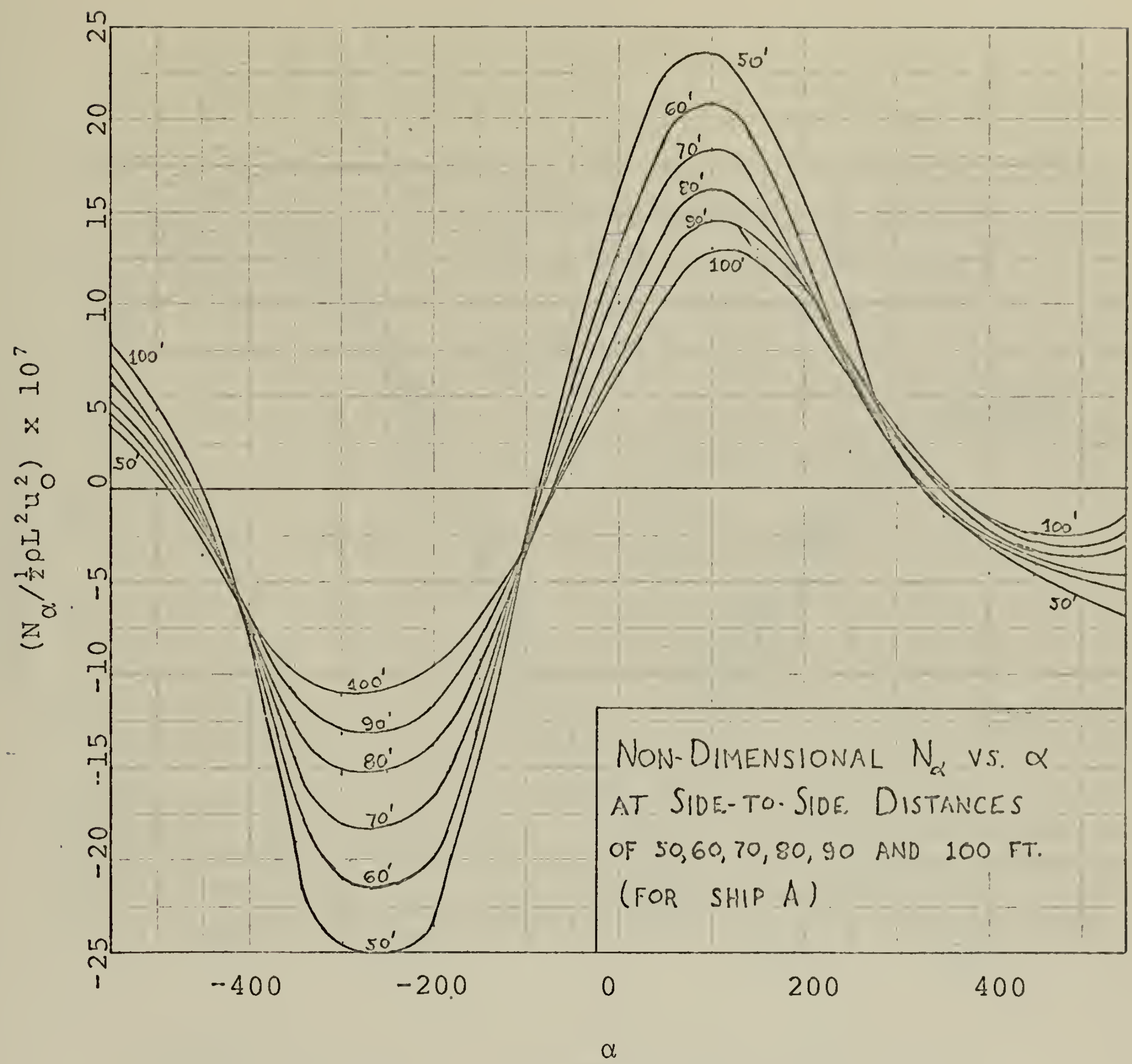
B. <u>Y_u, N_u.</u>

As noted in Chapter I, experimental investigators in the field of ship interactions have noted that interaction phenomena vary with speed in the same way as the resistance of the ship---or that interaction effects vary as the square of the speed. The OLNA data used for interaction effects was



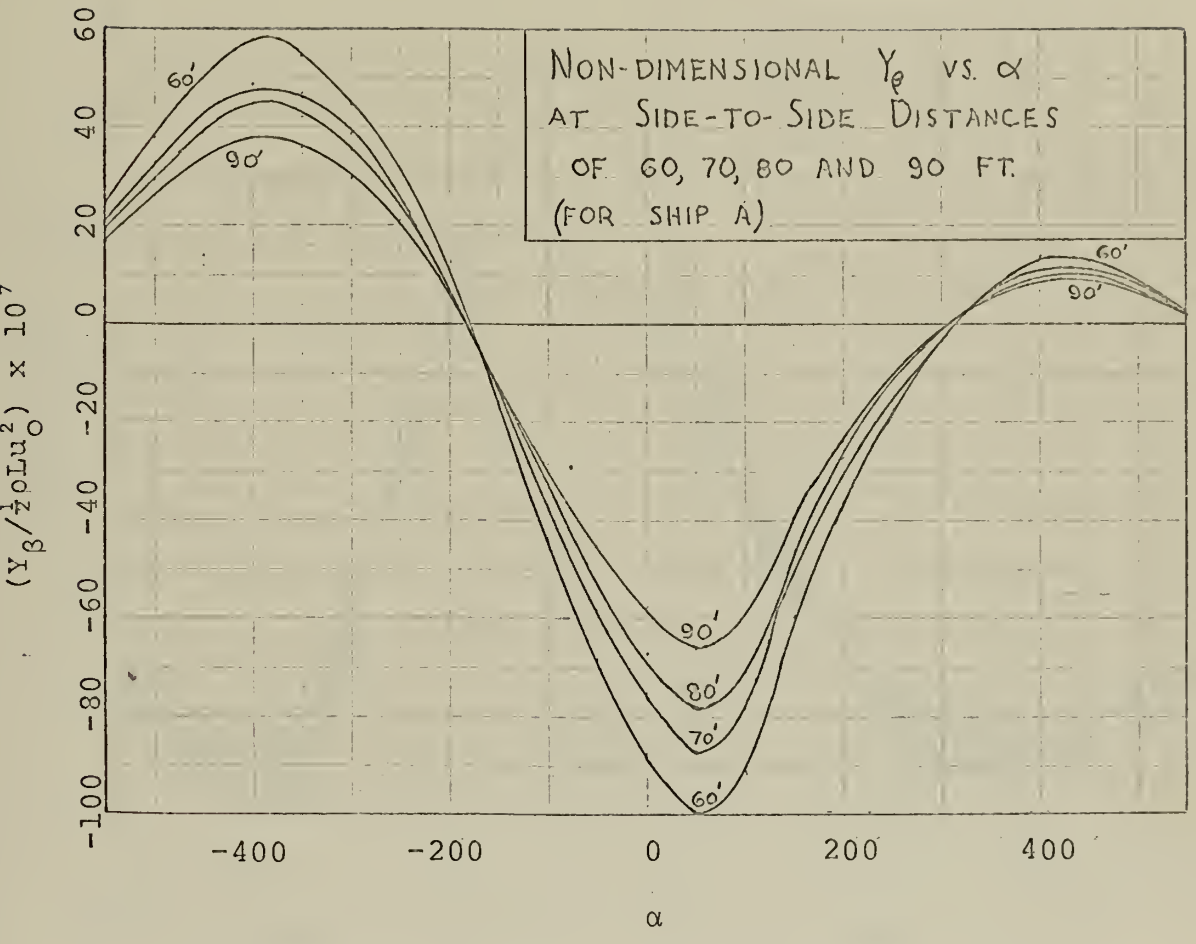
Dimensionless Y_α vs. Longitudinal Separation

FIGURE III-5



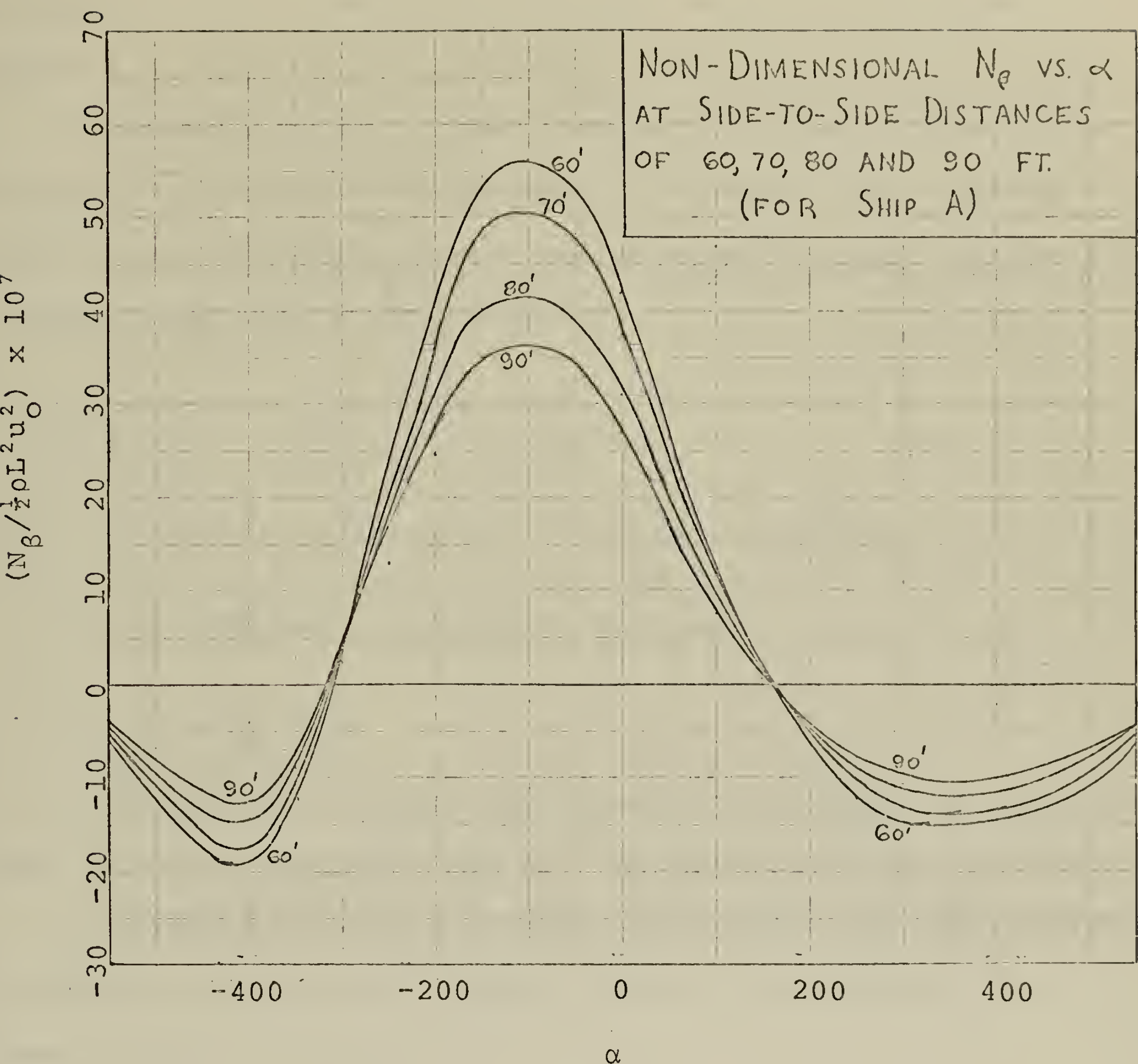
Dimensionless N_α vs. Longitudinal Separation

FIGURE III-6



Dimensionless Y_β vs. Longitudinal Separation

FIGURE III-7



Dimensionless N_β vs. Longitudinal Separation

FIGURE III-8

obtained at a speed of 15 knots, which was also used as the equilibrium speed for the results presented in this thesis. (However, all force, moment and derivative values are presented in non-dimensional form and are valid at other speeds.)

Remembering that Y_u, for example, is $\partial Y/\partial u$ with all other variables at equilibrium values, and knowing Y for a given position at 15 knots, it is a simple matter to get Y at 14 and 16 knots, and then Y_u at 15 knots.

a) $Y(16) = Y(15) \times (16/15)^2$

b) $Y(14) = Y(15) \times (14/15)^2$

c) $Y_u(15) = (Y(16)-Y(14))/((16-14)\times(1.6878))$ (3-4)

N_u is obtained by a similar process.

C. $\underline{X_n, Y_n, N_n}$.

The effects on forces and moments of a change in propeller speed are also required, and are represented by X_n, Y_n, and N_n.

The derivative X_n is determined by using the known values of Effective Horsepower, (EHP), and RPM for a MARINER at speeds around 15 knots.

a) $X(14kts) = (EHP(14)x550)/(15x1.6878)$

b) $X(16) = (EHP(16)x550)/(16x1.6878)$

c) $X_n(15) = (X(16)-X(14))/((RPM(16)-RPM(14))/60)$ (3-5)

The value of X_n so obtained is in dimensional form, and is then non-dimensionalized to permit broader applicability.

The single screw MARINER requires a 1.2° port rudder angle to maintain a steady course in open water. [A-4] Since the value of Y_δ for the MARINER is known from reference A-4, it is possible to determine what side force is being generated by this 1.2° rudder angle, which is denoted by $(\delta_o)_{ow}$. When maintaining a straight course in open water, this rudder side force is offsetting the side force caused by the asymmetry of a single screw. Thus, on a steady course, $Y_{rudder} = -Y_{prop}$.

The dimensionless Y_δ given in [A-4] can be converted to its dimensional form, which is in units of lbs/radian. The actual Y force exerted can then be calculated by multiplying the dimensional Y_δ by $(\delta_o)_{ow}/57.3°$, giving a side force in pounds which is the negative of the force being caused by the propeller. The force exerted by the propeller can be expected to vary as the propeller speed squared, and, thus, knowing the propeller side force at a given speed, it is possible to determine how this force varies as propeller speed, n, varies.

a) $Y_{rudder} = Y_\delta \text{xNDF}(@15 \text{ kts}) \times (\delta_o)_{ow}/(1 \text{ radian})$

b) $Y_{prop}(15 \text{ kts}) = -Y_{rudder}(15)$

c) $Y_{prop}(14) = Y_{prop}(15) \times (n(14)/n(15))^2$

d) $Y_{prop}(16) = Y_{prop}(15) \times (n(16)/n(15))^2$

e) $Y_n(15) = (Y_{prop}(16) - Y_{prop}(14))/(n(16) - n(14))$

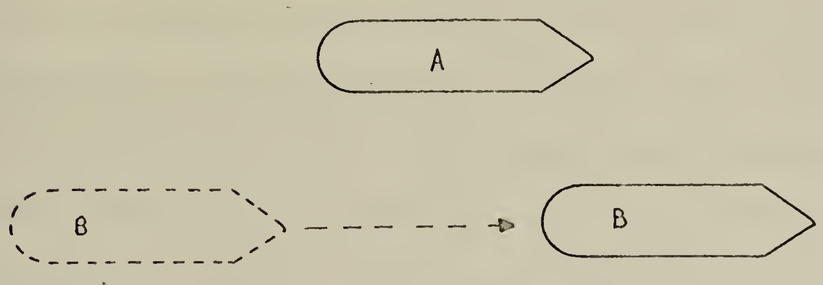
(Where NDF is the non-dimensionalizing factor for Y_δ)

The value of N_n is similarly obtained, and the Y_n and N_n so gotten are suitably non-dimensionalized.

A computer program, program DETERM, was written to determine the values of Y_u, N_u, X_n, Y_n and N_n. The inputs for the program are the values of Y and N at the positions of interest, gotten from program INTCOF, and the EHP, RPM, speed and $(\delta_o)_{ow}$ values for the ship being examined. The values of Y_δ and N_δ in open water are also inputs. Program DETERM is described in detail in Appendix III.

IV. Calculation of Interaction Derivatives for Ship B.

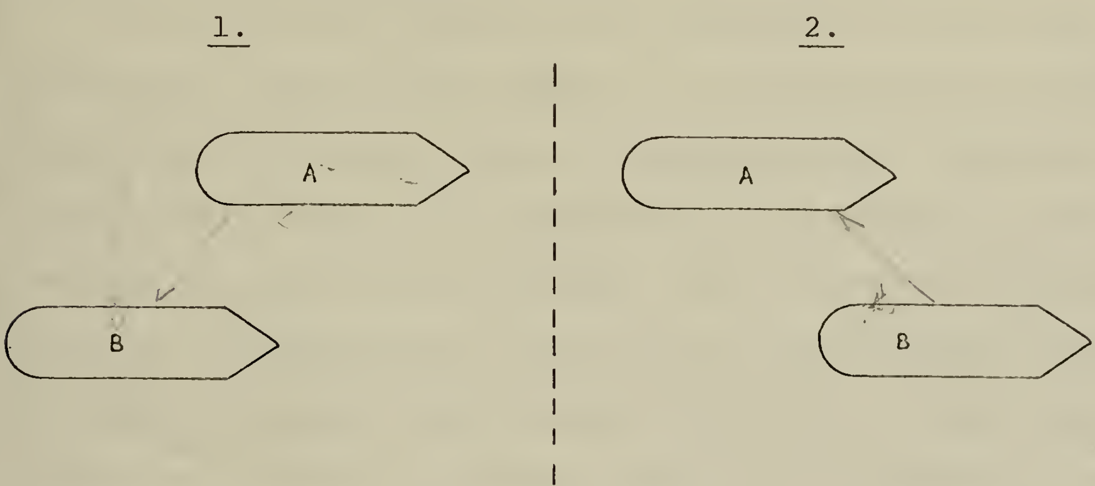
All discussion concerning interaction hydrodynamic derivatives, so far, has been for the OLNA/MARINER ship which is assumed to be Ship A of the two ship system. Ship A serves as the reference for measuring both longitudinal and lateral separations, α and β, and is assumed to be the overtaken ship, located to the left of Ship B, the overtaking ship.



This coincides with the case for the OLNA experiments and all plots of the interaction variables vs. α are for the overtaken ship on the left.

But hydrodynamic derivatives for both ships, at any relative position of interest, are needed to write the equations of motion for the system. Since the modified OLNA/

MARINER used as Ship A is the only one for which it was possible to obtain all required derivatives, Ship B must also be considered to be this type of ship, in order to be able to obtain derivatives for it. If the two ships are identical, the force or moment felt by Ship A at a given α and β are the negatives of the force or moment felt by Ship B at the same β at $\alpha = -\alpha$. To illustrate:



Y on Ship B at 2 equals -Y on Ship A at 1. If Ship A is attracted to B in position 1, then B will be attracted to A in the same way at 2. But because of the reference system, (as illustrated in Figure II-2), an attraction for Ship A is a positive Y force, while for Ship B an attraction is a negative Y force.

Thus, in program DETERM, the Y and N values entered for Ship B are the negatives of those for Ship A when $\alpha = -\alpha$. As an example, with STSD = 70' and α = 300', the non-dimensional Y for Ship A is $.1652 \times 10^{-3}$. The Y value entered for Ship B in this case is $-.1689 \times 10^{-3}$, which would be the value for Ship A when α = -300'. (These values are from the INTCOF

output data of Appendix II.) With the inputs so adjusted, program DETERM will calculate correct values of Y_u and N_u for both ships. (The calculations for X_n, Y_n, and N_n are independent of these considerations). The output of program DETERM is, therefore, X_n, Y_n, and N_n for Ship A, (which are assumed to be the same for Ship B), Y_u and N_u for Ship A and Y_u and N_u for Ship A, if it were in Ship B's position.

To bypass the necessity for considering both ships identical, provision was made to let Ship B be of a length different from Ship A, but of the same form. All form coefficients, plus the length/beam and beam/draft ratios, are assumed the same for both ships. This assumption implies that the non-dimensional values of the hydrodynamic derivatives for Ship B are the same as those for Ship A, (with some sign differences, as in Y_β and N_β), when all derivatives are non-dimensionalized with respect to the length of the ship to which they belong. But for the two ship system, the characteristic length used in the non-dimensionalizing process must be the same for <u>all</u> derivatives. It was decided to use the length between perpendiculars, LBP, of Ship A. Thus, if Ship B is of a length different than that of Ship A, the dimensionless derivatives of Ship A must be multiplied by (LBP_B/LBP_A) to get the dimensionless derivative for B with LBP_A replacing LBP_B in the non-dimensionalizing factor.

This modification of the Ship A derivatives to fit Ship B is done in the early part of a third computer program, SOLVE. (This program is used to actually solve the equations of

motion for the stability roots, as outlined below, but modifies the derivatives, as stated, before solving the equations). SOLVE is described in Appendix IV.

V. The Complete List of the Hydrodynamic Derivatives

Table III-1 lists the derivatives from reference A-4 for the MARINER and these are the values used for the open water derivatives for Ship A, and which are modified to apply to Ship B in program SOLVE.

Table III-2 lists the derivatives associated with propeller speed which were calculated by program DETERM for Ship A. In program SOLVE, they are used for Ship A and modified for Ship B.

The values of Y_α, Y_β, N_α, N_β, Y_u and N_u are position-dependent, since the values of Y and N acting as a result of interaction effects vary as the ships change positions relative to each other. In addition, these derivatives, even if ships A and B are identical, will not be the same for the two ships. Table III-3 gives values of these derivatives for the two ships, assuming them identical, for two typical positions of the ships relative to each other.

TABLE III-1

Derivative	Non-Dimensionalizing Factor	Non-Dimensional Value
$(X_{\dot{u}}-m)$	$1/2\rho L^3$	$-840.x10^{-5}$
X_u	$1/2\rho L^2 u_o$	$-120.x10^{-5}$
Y_v	$1/2\rho L^2 u_o$	$-1160.4x10^{-5}$
$(Y_{\dot{v}}-m)$	$1/2\rho L^3$	$-1546.x10^{-5}$
$(Y_{\dot{\psi}}-mu_o)$	$1/2\rho L^3 u_o$	$-499.x10^{-5}$
$(Y_{\ddot{\psi}}-mx_g)$	$1/2\rho L^4$	$8.6x10^{-5}$
Y_δ	$1/2\rho L^2 u_o^2$	$277.9x10^{-5}$
N_v	$1/2\rho L^3 u_o^2$	$-263.5x10^{-5}$
$(N_{\dot{v}}-mx_g)$	$1/2\rho L^4$	$22.7x10^{-5}$
$(N_{\dot{\psi}}-mx_g u_o)$	$1/2\rho L^4 u_o$	$-166.x10^{-5}$
$(N_{\ddot{\psi}}-I_z)$	$1/2\rho L^5$	$-82.9x10^{-5}$
N_δ	$1/2\rho L^3 u_o^2$	$-138.8x10^{-5}$

ρ = mass density of sea water 1.9905 lb-sec^2/ft^4

L = LBP Ship A, in ft

u_o = equilibrium speed in ft/sec

TABLE III-2

Derivative	Non-Dim. Factor	Non-Dim. Value
X_n	$1/2\rho L^3 u_o$	$4.62x10^{-5}$
Y_n	$1/2\rho L^3 u_o$	$-.52x10^{-5}$
N_n	$1/2\rho L^4 u_o$	$.26x10^{-5}$

TABLE III-3

α=0'; STSD=60'			
Deriv.	Non-Dim. Factor	Ship A Non-Dim. Value	Ship B Non-Dim. Value
Y_u	$1/2\rho L^2 u_o$	$152.5\text{x}10^{-5}$	$-152.5\text{x}10^{-5}$
N_u	$1/2\rho L^3 u_o$	$-67.1\text{x}10^{-5}$	$67.1\text{x}10^{-5}$
Y_α	$1/2\rho L\ u_o^2$	$.16\text{x}10^{-5}$	$.16\text{x}10^{-5}$
Y_β	$1/2\rho L\ u_o^2$	$-.87\text{x}10^{-5}$	$.87\text{x}10^{-5}$
N_α	$1/2\rho L^2 u_o^2$	$.126\text{x}10^{-5}$	$.126\text{x}10^{-5}$
N_β	$1/2\rho L^2 u_o^2$	$.42\text{x}10^{-5}$	$-.42\text{x}10^{-5}$
α= -400'; STSD=80'			
Y_u	Same	$-47.6\text{x}10^{-5}$	$-4.29\text{x}10^{-5}$
N_u		$17.3\text{x}10^{-5}$	$-16.7\text{x}10^{-5}$
Y_α	as	$.044\text{x}10^{-5}$	$-.093\text{x}10^{-5}$
Y_β	Above	$.447\text{x}10^{-5}$	$-.111\text{x}10^{-5}$
N_α		$-.068\text{x}10^{-5}$	$-.035\text{x}10^{-5}$
N_β		$-.14\text{x}10^{-5}$	$.098\text{x}10^{-5}$

VI. Method of Solving for Stability Roots

In Chapter II, it was shown that the stability characteristic equation for the system was obtained by evaluating the determinant formed by the coefficients of the variables. These coefficients are polynominals in $\mathcal{D}$, and it was stated that the operator could be treated as an algebraic quantity,

so that the result of the determinant evaluation was another polynominal in β, whose roots were the stability roots.

The coefficient determinant to be evaluated is a 6 x 6 determinant, given as equation (2-17). The determinant is evaluated by pivotal condensation ([A-13], pg. 155, problem 4-21). An illustration of pivotal condensation follows:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = a_{11} \times \begin{bmatrix} 1 & \frac{a_{12}}{a_{11}} & \cdots & \frac{a_{1n}}{a_{11}} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$= a_{11} \times \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & a'_{22} & \cdots & a'_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a'_{n2} & \cdots & a'_{nn} \end{bmatrix} = a_{11} \times \begin{bmatrix} a'_{22} & \cdots & a'_{2n} \\ \vdots & & \vdots \\ a'_{n2} & \cdots & a'_{nn} \end{bmatrix} \qquad (3\text{-}7)$$

Where $a'_{ij} = a_{ij} - (a_{1i}/a_{11})(a_{i1})$

The procedure would then be repeated with $\begin{bmatrix} a'_{22} & \cdots & a'_{2n} \\ \vdots & & \vdots \\ a'_{n2} & \cdots & a'_{nn} \end{bmatrix}$

The matrix properties whereby the interchanging of two rows or two columns changes the sign of the determinant are also utilized. After three applications of pivotal condensation and use of these two matrix properties, the determinant

in (2-17) becomes:

$$(AX1)(BX3')(BN6) \times \begin{vmatrix} AY4 & AY5 & AY2 \\ BY4' & BY5' & BY2' \\ AN4 & AN5 & AN2 \end{vmatrix} \qquad (3\text{-}8)$$

Where: $BX3' = BX3 - BX1(AX3/AX1)$

$BY4' = BY4 - BY6(BN4/BN6)$

$BY5' = BY5 - BY6(BN4/BN6)$

$BY2' = BY2 - BY6(BN4/BN6)$ (3-9)

Carrying the evaluation of (3-8) to its conclusion, and utilizing the relationships of (3-9), the determinant becomes:

$$\begin{aligned} DET = {} & [(AX1)(BX3)-(BX1)(AX3)] \\ & \times\{[(BN6)(BY5)-(BY6)(BN5)]\times[(AY4)(AN2)-(AN4)(AY2)] \\ & + [(BN6)(BY2)-(BY6)(BN2)]\times[(AY5)(AN4)-(AN5)(AY4)] \\ & + [(BN6)(BY4)-(BY6)(BN4)]\times[(AY2)(AN5)-(AN2)(AY5)]\} \end{aligned} \qquad (3\text{-}10)$$

Where the various terms are defined in Table III-4.

Program SOLVE, after appropriately modifying all hydrodynamic derivatives to apply to Ship B as well as Ship A, then calls a subroutine, DETER, to carry out the mathematical operations to evaluate (3-10) at one or more positions and with one or more sets of control system constants for the two ships at the various positions. Each of the terms in (3-10), AX2, BN6, AY2, etc., is a polynomial in $\mathcal{D}$, and the various multiplications, subtractions and additions are carried out in subroutine DETER by using IBM provided Scientific Subroutine

$$AX1 = X_{n_A} k_{5_A} + X_{n_A} (k_{6_A} - k_{5_A} \Delta t_{2_A}) \mathcal{D} - X_{n_A} k_{6_A} \Delta t_{2_A} \mathcal{D}^2$$

$$AX3 = X_{u_A} + (X_{\dot{u}_A} - m_A) \mathcal{D}$$

$$AY2 = (Y_{\beta_A} + Y_{\delta_A} k_{3_A}) + Y_{\delta_A} (k_{4_A} - k_{3_A} \Delta t_{1_A}) \mathcal{D} - Y_{\delta_A} k_{4_A} \Delta t_{1_A} \mathcal{D}^2$$

$$AY4 = Y_{v_A} + (Y_{\dot{v}_A} - m_A) \mathcal{D}$$

$$AY5 = Y_{\delta_A} k_{1_A} + ((Y_{\dot{\psi}_A} - m_A u_o) + Y_{\delta_A} (k_{2_A} - k_{1_A} \Delta t_{1_A})) \mathcal{D} + ((Y_{\ddot{\psi}_A} - m_A x_{g_A}) - Y_{\delta_A} k_{2_A} \Delta t_{1_A}) \mathcal{D}^2$$

$$AN2 = (N_{\beta_A} + N_{\delta_A} k_{3_A}) + N_{\delta_A} (k_{4_A} - k_{3_A} \Delta t_{1_A}) \mathcal{D} - N_{\delta_A} k_{4_A} \Delta t_{1_A} \mathcal{D}^2$$

$$AN4 = N_{v_A} + (N_{\dot{v}_A} - m_A x_{g_A}) \mathcal{D}$$

$$AN5 = N_{\delta_A} k_{1_A} + ((N_{\dot{\psi}_A} - m_A x_{g_A} u_o) + N_{\delta_A} (k_{2_A} - k_{1_A} \Delta t_{1_A})) \mathcal{D} + ((N_{\ddot{\psi}_A} - I_{z_A}) - N_{\delta_A} k_{2_A} \Delta t_{1_A}) \mathcal{D}^2$$

$$BX1 = X_{n_B} k_{5_B} + (X_{n_B} (k_{6_B} - k_{5_B} \Delta t_{2_B}) + X_{u_B}) \mathcal{D} + ((X_{\dot{u}_B} - m_B) - X_{n_B} k_{6_B} \Delta t_{2_B}) \mathcal{D}^2$$

$$BX3 = X_{u_B} + (X_{\dot{u}_B} - m_B) \mathcal{D}$$

$$BY2 = (Y_{\beta_B} + Y_{\delta_B} k_{3_B}) + (Y_{\delta_B} (k_{4_B} - k_{3_B} \Delta t_{1_B}) + Y_{v_B}) \mathcal{D} + ((Y_{\dot{v}_B} - m_B) - Y_{\delta_B} k_{4_B} \Delta t_{1_B}) \mathcal{D}^2$$

$$BY4 = Y_{v_B} + (Y_{\dot{v}_B} - m_B) \mathcal{D}$$

$$BY5 = u_o Y_{v_B} + u_o (Y_{\dot{v}_B} - m_B) \mathcal{D}$$

$$BY6 = (Y_{\delta_B} k_{1_B} - u_o Y_{v_B}) + ((Y_{\dot{\psi}_B} - m_B u_o) + Y_{\delta_B}(k_{2_B} - k_{1_B} \Delta t_{1_B}) - u_o (Y_{\dot{v}_B} - m_B))\mathcal{D} + ((Y_{\ddot{\psi}_B} - m_B x_{g_B}) - Y_{\delta_B} k_{2_B} \Delta t_{1_B})\mathcal{D}^2$$

$$BN2 = (N_{\beta_B} + N_{\delta_B} k_{3_B}) + (N_{\delta_B}(k_{4_B} - k_{3_B} \Delta t_{1_B}) + N_{v_B})\mathcal{D} + ((N_{\dot{v}_B} - m_B x_{g_B}) - N_{\delta_B} k_{4_B} \Delta t_{1_B})\mathcal{D}^2$$

$$BN4 = N_{v_B} + (N_{\dot{v}_B} - m_B x_{g_B})\mathcal{D}$$

$$BN5 = u_o N_{v_B} + \dot{u}_o (N_{\dot{v}_B} - m_B x_{g_B})\mathcal{D}$$

$$BN6 = (N_{\delta_B} k_{1_B} - u_o N_{v_B}) + ((N_{\dot{\psi}_B} - m_B x_{g_B} u_o) + N_{\delta_B}(k_{2_B} - k_{1_B} \Delta t_{1_B}) - u_o (N_{\dot{v}_B} - m_B x_{g_B}))\mathcal{D}$$
$$+ ((N_{\ddot{\psi}_B} - I_{z_B}) - N_{\delta_B} k_{2_B} \Delta t_{1_B})\mathcal{D}^2$$

Definition of Terms in Determinant

TABLE III-4

Package (SSP) subroutines for arithmetic operations with polynomials. Program SOLVE, (and all the subroutines it calls), is described in detail in Appendix IV.

VII. Control Constant Magnitude

Reference A-11 discusses automatic ship control and utilizes some typical values for the control constants in a system sensitive to ψ and $\dot{\psi}$. Sensitivity to an error parameter, such as ψ, is proportional control, and to an error parameter rate, such as $\dot{\psi}$, is rate control. The equation for rudder control in such a system, (not considering signs), is:

$$\delta = k_1\psi + k_2\dot{\psi} \tag{3-11}$$

In [A-11], Schiff and Gimprich use values of k_1 between 1.0 and 5.0, as typical values. The only values used for k_2 are 0 or 1.0.

With $k_1 = 1.0$, a 1° heading error will give rise to 1° of correcting rudder, while with $k_1 = 5.0$, a 1° error will cause a 5° rudder correction. If $k_2 = 1.0$, a heading error rate of 1°/sec. gives a rudder angle of 1° for correction. Thus, k_1 was considered to vary from 1.0 to 5.0, (with smaller values investigated to determine some trends), and k_2 was varied between 0.5 and 4. (Defining δ and ψ to be in radian does not affect k_1 and k_2).

In determining ranges to be used for k_3 and k_4, which determine sensitivity of the control system to lateral separation error and its rate of change, β and $\dot{\beta}$, no basis

was found in the literature. Thus, these constants were estimated from what the author expected to be reasonable corrections of the rudder angle for lateral separation errors. Rudder angle is sensitive to β as below, (again signs are not considered):

$$\delta = k_3\beta + k_4\dot{\beta} \tag{3-12}$$

Based on the author's experience in underway replenishment operations, it was assumed that an error of 10 feet in β would prompt a rudder correction of about 1°. And a change in $\dot{\beta}$ of 1 ft/sec. was felt to call for 3° of rudder correction. Therefore:

$$k_3 \approx \frac{1°}{10\text{ft}} \times \frac{1 \text{ radian}}{57.3°} \approx \frac{1}{57.3} \approx .0017$$

$$k_4 \approx \frac{3°}{1\text{ft/sec}} \times \frac{1 \text{ radian}}{57.3°} \approx \frac{3}{57.3} \approx .052 \tag{3-13}$$

As a result, k_3 was investigated between .0005 and .01 and k_4 between .01 and .1.

A propeller speed control, sensitive to longitudinal separation error and its rate, α and $\dot{\alpha}$, was assumed in the theory. For this sensitivity, not considering signs:

$$n = k_5\alpha + k_6\dot{\alpha} \tag{3-14}$$

To get an idea of the magnitude of k_5 and k_6, estimates were resorted to. It was assumed that a typical error of 10 feet in α would prompt an n change of 1 RPM, (about 1/4 knot for a MARINER) and a 1 ft/sec. value for $\dot{\alpha}$ would prompt a change of 4 RPM. Therefore:

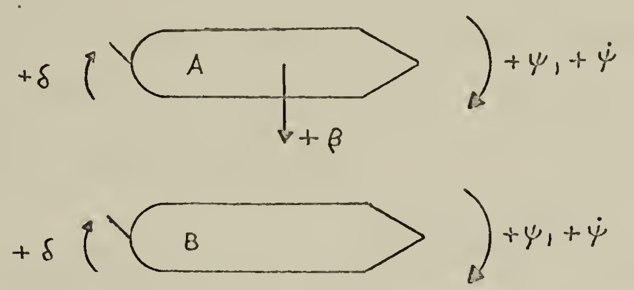
$$k_5 \approx \frac{1\ \text{RPM}}{10\ \text{ft}} \times \frac{1}{60\text{RPM/RPS}} \approx \frac{1}{600} \approx .0016$$

$$k_6 \approx \frac{4\ \text{RPM}}{1\text{ft/sec}} \times \frac{1}{60\text{RPM/RPS}} \approx \frac{1}{15} \approx .066 \qquad (3\text{-}15)$$

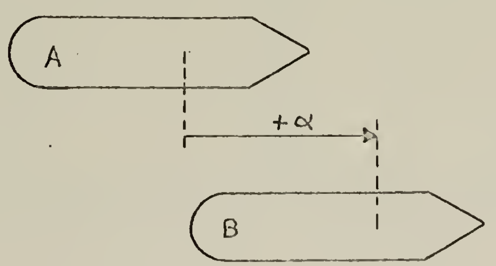
So k_5 was assumed to be between .0005 and .01 in value, and k_6 between .01 and .2.

VIII. Control Constant Signs

When either ship develops a positive error in ψ or $\dot{\psi}$, a positive rudder angle is required to correct for the error. Thus k_1 and k_2 are positive for both ships.



As β or $\dot{\beta}$ increase in a positive direction, it is desired to turn Ship B to the left and Ship A to the right, to compensate. Thus as β increases, a negative δ is required on Ship A, but a positive δ is wanted for Ship B. Therefore k_3 and k_4 are negative for Ship A and positive for Ship B.



As α increases (or $-\alpha$ decreases in magnitude), it is desired to increase Ship A's speed and decrease Ship B's. Therefore k_5 and k_6 are positive for Ship A and negative for Ship B.

Both rudder and propeller response will have time lags associated with them. The rudder, however, can be expected to respond more quickly than propeller speed. Thus rudder time lags from 0 to 4 seconds, and propeller time lags from 0 to 6 seconds are examined. (The desire is to determine the way in which time lags affect stability, and errors in the actual values used will not affect trends developed.)

To summarize, Table III-5 lists all the control system parameter ranges examined.

TABLE III-5

Ship A	Ship B
$1. \leq k_1 \leq 5.$	$1. \leq k_1 \leq 5.$
$.5 \leq k_2 \leq 4.$	$.5 \leq k_2 \leq 4.$
$-.01 \leq k_3 \leq -.0005$	$.0005 \leq k_3 \leq .01$
$-.1 \leq k_4 \leq -.01$	$.01 \leq k_4 \leq .1$
$.0005 \leq k_5 \leq .01$	$-.01 \leq k_5 \leq -.0005$
$.01 \leq k_6 \leq .2$	$-.2 \leq k_6 \leq -.01$
$0 \leq \Delta t_1 \leq 4.$	$0 \leq \Delta t_1 \leq 4.$
$0 \leq \Delta t_2 \leq 6.$	$0 \leq \Delta t_2 \leq 6.$

Chapter IV

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

I. Introduction

The intent of this thesis is to develop and solve the equations of motion for the two ship system, and to obtain the stability roots for it. The method, with the computer programs used to perform the calculations, (which would be prohibitive by hand), permits determination of whether or not the system is stable for a given set of control constants and time lags at given positions. In order to examine the accuracy of the solutions and correctness of the method, various sets of control system constants, for various ship positions, were used as inputs to the computer programs and the effect on the stability of the system observed.

It is beyond the scope of this thesis to attempt an optimization of, or even complete examination of, the effects of variations in the automatic control parameters upon system stability. The purpose is to present the method, with only enough calculation to assess the value of the theoretical formulation and computer solution. As a result, the various combinations of controls and ship positions examined are by no means exhaustive---but are representative.

II. Results

A few sets of control constants were evaluated at each of four positions, for ships of different size, with and without

time lag, and are presented in Appendix IV as typical output from program SOLVE.

To perform a somewhat more complete examination of system stability, a single position, (α = 0', STSD = 90'), was chosen, and a total of 91 combinations of control constants and time lags was run. The results of these runs are summarized in Tables IV-1, IV-2, and IV-3. In these tables, the number of zero roots, number of negative roots and number of positive roots obtained for the input control constants are indicated. In addition, the largest root for each condition is listed, unless the largest root is zero. In this case the largest negative root is listed. The effects of changes in some of the control constants on the value of the largest root are shown in Figures IV-1 through IV-5.

III. Discussion of Results

The first part of Table IV-1 summarizes the effect of varying k_1, with all other control parameters zero and no time lags. These results are illustrated in Figure IV-1. Varying k_{1_A}, alone, from a very low value to its maximum, decreases instability for a time, (the value of the largest root is positive, but decreasing), but then instability increases as k_{1_A} increases. When k_{1_A} and k_{1_B} are both increased through this same range, the system instability is lessened, with stability actually acheived for part of the range, but the trend for instability to increase for the higher k_1 values is still present.

The system investigated to get the results presented in this chapter consisted of two identical ships. This permitted a check of the method to see that the solution is symmetrical for this symmetrical system. As a check, a run was made with ${}^{k}1_B = 3$ and ${}^{k}1_A = 0$, which yielded results identical to those when ${}^{k}1_A = 3$ and ${}^{k}1_B = 0$. Thus the system of equations, and their solution, appear to be correct in at least this respect.

The second and third parts of Table IV-1 summarize the results of varying k_3 and k_5 for the two ships, with all other constants zero. As can be seen from the table, and from Figure IV-2, neither k_3 nor k_5, alone, even when present on both ships, is sufficient to bring about system stability. As k_3 is increased, for one or both ships, with all other k's zero, system instability is actually increased. And as k_5 is increased, the value of the largest root is unaffected, though the values of the other roots do change slightly. (The symmetry of the solution was again demonstrated for these constants.)

Next, ${}^{k}3_A$, ${}^{k}3_B$, ${}^{k}5_A$ and ${}^{k}5_B$ were all given values in their expected ranges, while ${}^{k}1_A$ and ${}^{k}1_B$ were varied. The results are summarized in the fourth part of Table IV-1 and illustrated in Figure IV-3. In these circumstances, increasing ${}^{k}1_A$ and ${}^{k}1_B$ reduces system instability until a stable condition is attained.

TABLE IV-1

Summary of Stability Roots Obtained for Various Combinations of Control System Constants With No Time Lags

$\alpha = 90'$; STSD = 0'; V_o = 15 kts.

Ship A LBP = 528.5'; Ship B LBP = 528.5'

All unlisted $k_i = 0$

1.	k_{1_A}	k_{1_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. $\neq$ 0	Stable?
	0	0	2	6	2	$.1227\text{x}10^{-2}$	no
	.001	0	1	7	2	$.1201\text{x}10^{-2}$	no
	.01	0	1	7	2	$.9343\text{x}10^{-3}$	no
	.1	0	1	7	2	$.3293\text{x}10^{-3}$	no
	1.	0	1	7	2	$.6980\text{x}10^{-3}$	no
	2.	0	1	7	2	$.7186\text{x}10^{-3}$	no
	3.	0	1	7	2	$.7255\text{x}10^{-3}$	no
	4.	0	1	7	2	$.7289\text{x}10^{-3}$	no
	5.	0	1	7	2	$.7310\text{x}10^{-3}$	no
	0	3.	1	7	2	$.7255\text{x}10^{-3}$	no
	.001	.001	1	7	2	$.1175\text{x}10^{-2}$	no
	.01	.01	1	7	2	$.6861\text{x}10^{-3}$	no
	.1	.1	1	9	0	$-.2123\text{x}10^{-2}$	yes
	1.	1.	1	9	0	$-.1877\text{x}10^{-3}$	yes
	2.	2.	1	9	0	$-.9270\text{x}10^{-4}$	yes
	3.	3.	1	9	0	$-.6111\text{x}10^{-4}$	yes
	4.	4.	1	9	0	$-.4534\text{x}10^{-4}$	yes
	5.	5.	1	9	0	$-.3587\text{x}10^{-4}$	yes

TABLE IV-1
(con't)

2.	k_{3_A}	k_{3_B}	# 0 Rts.	#< 0 Rts.	#> 0 Rts.	Largest Rt. $\neq$ 0	Stable?
	-.005	0	2	6	2	$.2809\times10^{-1}$	no
	-.01	0	2	6	2	$.3720\times10^{-1}$	no
	-.03	0	2	6	2	$.5848\times10^{-1}$	no
	-.05	0	2	6	2	$.7242\times10^{-1}$	no
	-.07	0	2	6	2	$.8348\times10^{-1}$	no
	0	.01	2	6	2	$.3720\times10^{-1}$	no
	-.005	.005	2	6	2	$.3720\times10^{-1}$	no
	-.008	.008	2	6	2	$.4509\times10^{-1}$	no
	-.01	.01	2	6	2	$.4940\times10^{-1}$	no
	-.03	.03	2	6	2	$.7821\times10^{-1}$	no
	-.05	.05	2	6	2	$.9741\times10^{-1}$	no
	-.07	.07	2	6	2	.1122	no

TABLE IV-1
(con't)

3.	k_{5_A}	k_{5_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
	.005	0	1	7	2	$.1227\times10^{-2}$	no
	.008	0	1	7	2	"	no
	.01	0	1	7	2	"	no
	.03	0	1	7	2	"	no
	.05	0	1	7	2	"	no
	.07	0	1	7	2	"	no
	0	-.01	1	7	2	"	no
	.005	-.005	1	7	2	"	no
	.008	-.008	1	7	2	"	no
	.01	-.01	1	7	2	"	no
	.03	-.03	1	7	2	"	no
	.05	-.05	1	7	2	"	no
	.07	-.07	1	7	2	"	no

$k_{3_A} = -.005 = -k_{3_B}$; $k_{5_A} = .01 = -k_{5_B}$

4.	k_{1_A}	k_{1_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
	1.	1.	0	8	2	$.2982\times10^{-1}$	no
	2.	2.	0	8	2	$.2163\times10^{-1}$	no
	3.	3.	0	8	2	$.1245\times10^{-1}$	no
	4.	4.	0	8	2	$.2264\times10^{-2}$	no
	5.	5.	0	10	0	$-.3403\times10^{-2}$	yes

TABLE IV-1
(con't)

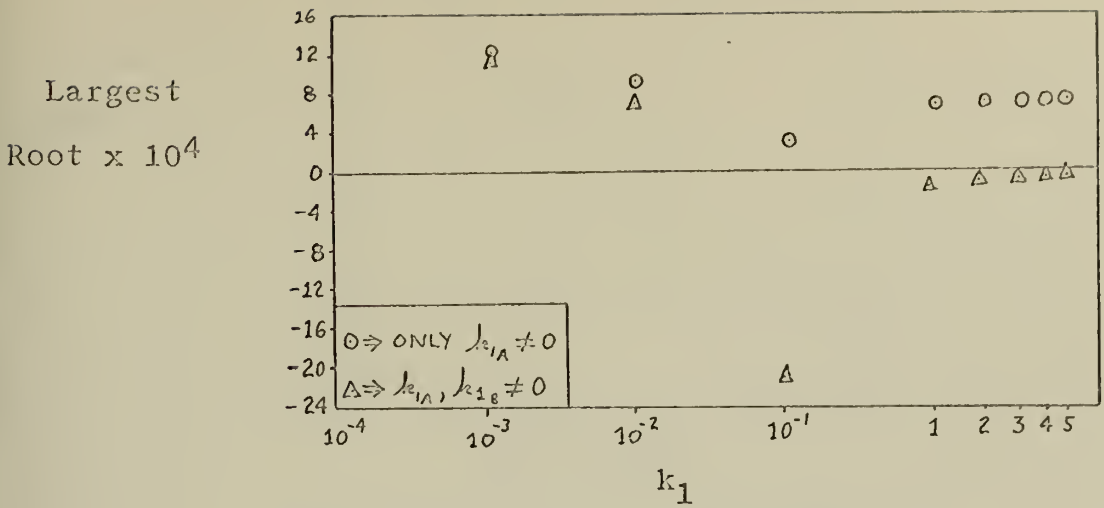
$k_{1_A} = k_{2_B} = 2.$; $k_{3_A} = -.005 = -k_{3_B}$; $k_{5_A} = .01 = -k_{5_B}$								
5.	k_{2_A}	k_{2_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
	.5	.5	0	8	2	$.2126\text{x}10^{-1}$	no
	1.	1.	0	8	2	$.2090\text{x}10^{-1}$	no
	2.	2.	0	8	2	$.2018\text{x}10^{-1}$	no
	3.	3.	0	8	2	$.1950\text{x}10^{-1}$	no
	4.	4.	0	8	2	$.1883\text{x}10^{-1}$	no
	k_{4_A}	k_{4_B}					
	-.01	.01	0	8	2	$.1996\text{x}10^{-1}$	no
	-.04	.04	0	8	2	$.1595\text{x}10^{-1}$	no
	-.07	.07	0	8	2	$.1427\text{x}10^{-1}$	no
	-.1	.1	0	8	2	$.1499\text{x}10^{-1}$	no
	k_{6_A}	k_{6_B}					
	.01	-.01	0	8	2	$.2163\text{x}10^{-1}$	no
	.05	-.05	0	8	2	"	no
	.1	-.1	0	8	2	"	no
	.2	-.2	0	8	2	"	no

TABLE IV-1
(con't)

$k_{1_A} = k_{1_B} = 2.;\ k_{3_A} = -.005 = -k_{3_B};\ k_{5_A} = .01 = -k_{5_B};$

$k_{2_A} = k_{2_B};\ k_{4_A} = -k_{4_B};\ k_{6_A} = -k_{6_B}$

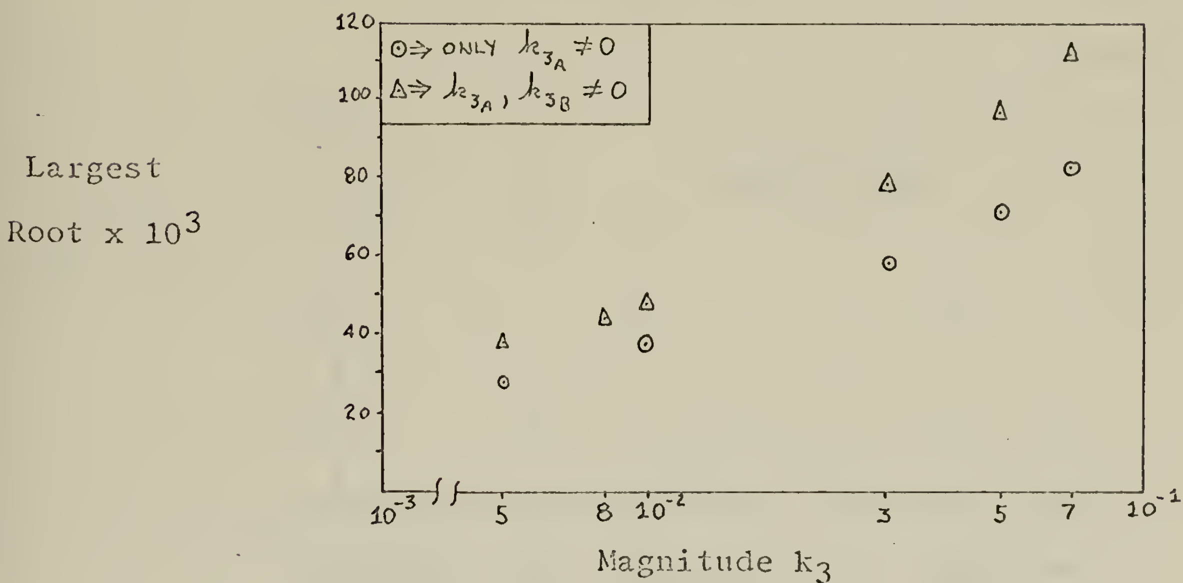
	k_2	$\lvert k_4\rvert$	$\lvert k_6\rvert$	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
6.	.5	.01	.01	0	8	2	$.1956\times10^{-1}$	no
	1.	.04	.05	0	8	2	$.1493\times10^{-1}$	no
	2.	.07	.1	0	8	2	$.1181\times10^{-1}$	no
	4.	.1	.2	0	8	2	$.9429\times10^{-2}$	no
	Same, but $k_{1_A} = k_{1_B} = 3.$							
7.	.5	.01	.1	0	8	2	$.1030\times10^{-1}$	no
	1.	.04	.1	0	8	2	$.6420\times10^{-2}$	no
	2.	.05	.1	0	8	2	$.4783\times10^{-2}$	no
	Same, but $k_{1_A} = k_{1_B} = 4.$							
8.	1.	.04	.1	0	10	0	$-.1449\times10^{-2}$	yes
	2.	.04	.1	0	10	0	$-.2810\times10^{-2}$	yes



k_1 vs. Largest Root

for all $\Delta t=0$

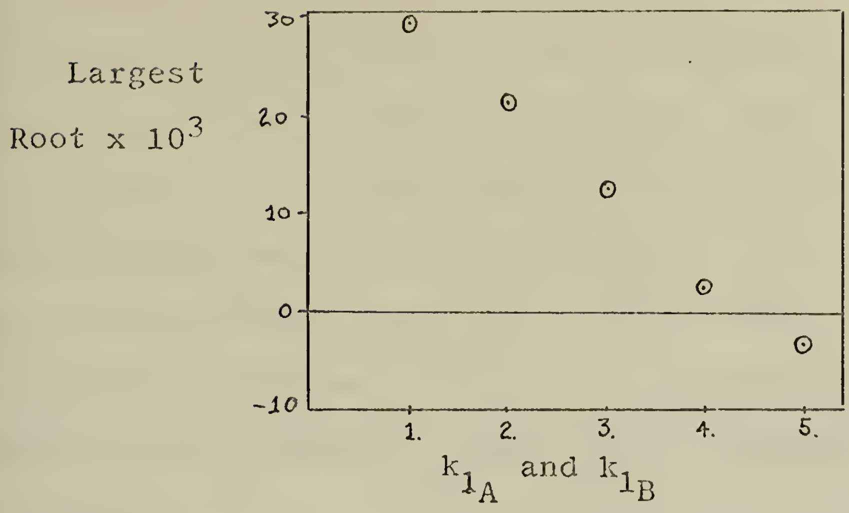
FIGURE IV-1



k_3 vs. Largest Root

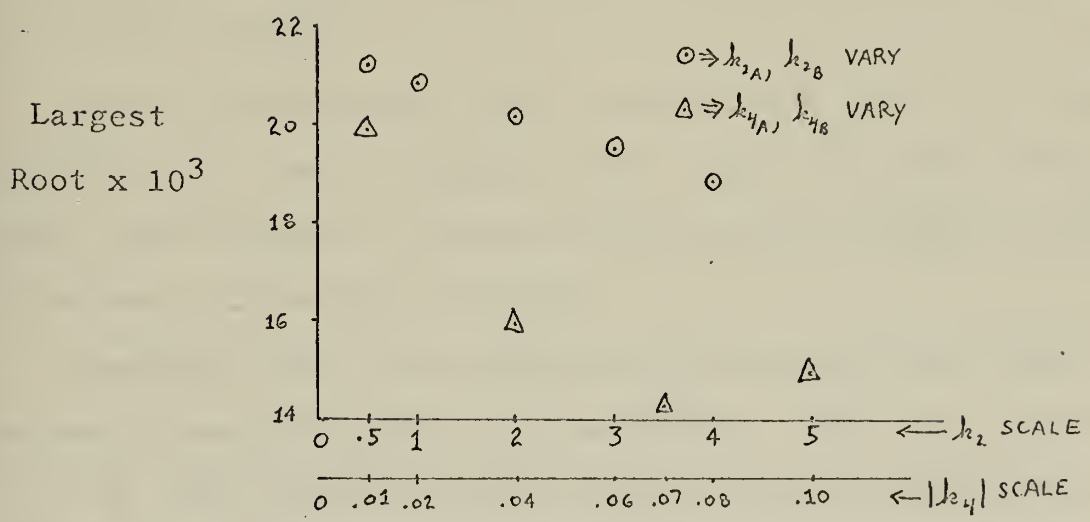
for all $\Delta t=0$

FIGURE IV-2



k_{1_A} and k_{1_B} vs. Largest Root
(k_{3_A}=-.005=-k_{3_B}; k_{5_A}=.01=-k_{5_B};
All Δt=0)

FIGURE IV-3



k_2 and $|k_4|$ vs Largest Root
(k_{1_A}=k_{1_B}=2.; k_{3_A}=-.005=-k_{3_B};
k_{5_A}=.01=-k_{5_B}; All Δt=0)

FIGURE IV-4

Then, with the proportional constants for both ships, (k_1, k_3 and k_5), set at values within their ranges, the rate constants, (k_2, k_4 and k_6), were systematically increased. This step is summarized in Table IV-1, part 5, and the effects for k_2 and k_4 are illustrated in Figure IV-4. As can be seen, increasing k_2 decreased system instability, and increasing k_4 had the same effect until $|k_4|$ reached about .07, after which instability increased. Varying k_6 had no effect on the largest root, though the other roots changed slowly as k_6 increased.

In part 6 of Table IV-1, the effect of increasing all three rate constants, with the proportional constants fixed, is shown to be a decrease in instability. And in parts 7 and 8, the same trend is shown with the k_1 values being set at first 3.0, then 4.0. This last combination is seen to result in a stable system.

Some investigations with non-zero time lags are summarized in Table IV-2, and in Figure IV-5. As can be seen, in all cases investigated an increase in time lags caused the system to become increasingly unstable.

In Table IV-3, the results of giving only one ship an automatic control system are summarized. For this case all $k_{i_B} = 0$. This table indicates that a stable system can be achieved with only one ship automatically controlled, with control constants in the expected ranges. Physically, this would appear to indicate that one ship has been "captured"

TABLE IV-2

Summary of Stability Roots Obtained for Various Combinations of Control System Constants With Time Lags

$\alpha = 90'$; STSD = 0'; V_o = 15 kts.

Ship A LBP = 528.5'; Ship B LBP = 528.5'

All unlisted $k_i = 0$

$k_{1_A} = k_{1_B} = 2.$; $k_{3_A} = -.005 = -k_{3_B}$; $k_{5_A} = .01 = -k_{5_B}$

Δt_{1_A}	Δt_{2_A}	Δt_{1_B}	Δt_{2_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
1	2	1	2	0	8	2	$.2399\times10^{-1}$	no
2	4	2	4	0	6	4	$.2637\times10^{-1}$	no
4	6	4	6	0	6	4	$.3116\times10^{-1}$	no

Same, plus $k_{2_A} = k_{2_B} = 2.$

1	2	1	2	0	8	2	$.2246\times10^{-1}$	no
2	4	2	4	0	6	4	$.2477\times10^{-1}$	no
4	6	4	6	0	6	4	$.2951\times10^{-1}$	no

Same, plus $k_{4_A} = -.04 = -k_{4_B}$

1	2	1	2	0	8	2	$.2027\times10^{-1}$	no
2	4	2	4	0	6	4	$.2452\times10^{-1}$	no
4	6	4	6	0	6	4	$.3278\times10^{-1}$	no

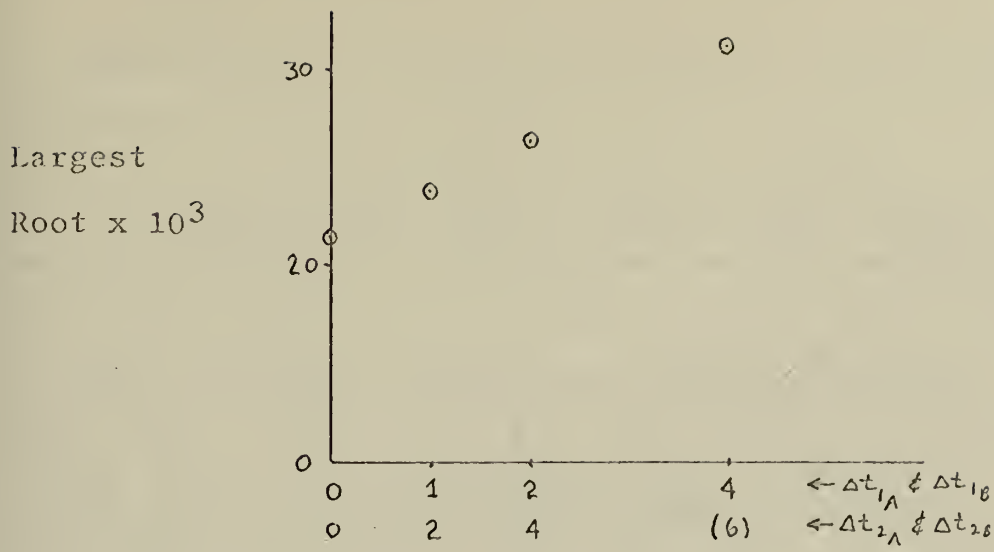
TABLE IV-2
(con't)

Same, plus $k_{6_A} = .1 = -k_{6_B}$

Δt_{1_A}	Δt_{2_A}	Δt_{1_B}	Δt_{2_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
1	2	1	2	0	8	2	$.2399 \times 10^{-1}$	no
2	4	2	4	0	8	2	$.2637 \times 10^{-1}$	no
4	6	4	6	0	8	2	$.3116 \times 10^{-1}$	no

Same, plus $k_{2_A} = k_{2_B} = 2.$; $k_{4_A} = -.04 = -k_{4_B}$; $k_{6_A} = .1 = -k_{6_B}$

Δt_{1_A}	Δt_{2_A}	Δt_{1_B}	Δt_{2_B}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
1	2	1	2	0	8	2	$.1780 \times 10^{-1}$	no
2	4	2	4	0	8	2	$.2362 \times 10^{-1}$	no
4	6	4	6	0	8	2	$.3473 \times 10^{-1}$	no



Time Lags vs. Largest Root

$(k_{1_A}=k_{1_B}=2.;\ k_{3_A}=-.005=-k_{3_B};$
$k_{5_A}=.01=-k_{5_B})$

FIGURE IV-5

TABLE IV-3

Summary of Stability Roots Obtained for Various Combinations of Single Ship Control With No Time Lags

$\alpha = 90'$; STSD = $0'$; $V_o = 15$ kts.

Ship A LBP = 528.5'; Ship B LBP = 528.5'

$k_{1_A} = 2.$; $k_{3_A} = -.005$; $k_{5_A} = .01$; All $k_{i_B} = 0$

k_{2_A}	k_{4_A}	k_{6_A}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
.5	-.01	.01	0	8	2	$.6793 \times 10^{-2}$	no
1.	-.04	.05	0	8	2	$.3092 \times 10^{-2}$	no
2.	-.07	.1	0	8	2	$.3914 \times 10^{-5}$	no
4.	-.1	.2	0	10	0	$-.2852 \times 10^{-3}$	yes

$k_{2_A} = 2.$; $k_{3_A} = -.005$; $k_{4_A} = -.05$; $k_{5_A} = .01$; $k_{6_A} = .1$; $k_{i_B} = 0$

k_{1_A}	# 0 Rts.	#<0 Rts.	#>0 Rts.	Largest Rt. ≠ 0	Stable?
3.	0	10	0	$-.4373 \times 10^{-3}$	yes
4.	0	10	0	$-.6003 \times 10^{-3}$	yes

by the other through the interaction force and moment. This was an unexpected result and it was felt that it might be indicative of an error in the formulation or solution of the equations.

To investigate this possibility, some extra computer runs, for special circumstances, were made. In these runs, all the interaction force, moment and derivative inputs for program SOLVE were set equal to zero. This is equivalent to the ships being infinitely far apart, where no interaction effects can be felt. In this circumstance, it should prove possible to obtain a stable system with reasonable control constant values for each ship, and, this, in fact, was the case.

A further check was made, where the ships were given control systems with different constants---then run again with the same two sets of control constants, but switched from one ship to the other---both runs made with the interaction effects all set at zero. These two runs should result in identical sets of stability roots if the solution is correctly done, and, did, in fact, do so.

The last check run to verify the accuracy of the solution method was with all control constants zero for both ships. The interaction effects were all left at zero, as well. This, in effect, puts the ships infinitely far apart, with no automatic control, and should result in neutral directional stability. The result of this run was four zero roots and

six negative roots, indicating a dynamically stable system, neutrally stable, probably with regard to heading

IV. Conclusions

The investigation reported herein has prompted the following conclusions:

A. The methods of formulation and solution of the equations of motion appear to be correct.

B. It appears possible to achieve a stable system of two ships on parallel courses in close proximity, using automatic control.

C. System stability is most affected by control sensitivity to heading angle error, ---that is, k_1 terms; and least affected by sensitivity to longitudinal position error and error rate, and ---that is, k_5 and k_6 terms.

D. It is possible to achieve a stable system with proportional control only, (k_2 k_4 k_6 0).

E. It is possible, in isolated circumstances, at least, to achieve a stable system with only one ship employing automatic control.

F. Under some circumstances, increasing the magnitude of a control constant, and thereby increasing system sensitivity to one of the error parameters, can have an adverse effect on system stability, (as illustrated in Figure IV-2).

V. Recommendations

The following recommendations are made for future investigation into the practicality of employing automatic controls for ships engaged in operations alongside and close aboard, as in underway replenishment:

A. A detailed examination of the theoretical development used in this thesis should be conducted by someone other than the author, to remove any reservation as to the accuracy of the method.

B. The examination of system stability should be made for two ships whose full set of hydrodynamic derivatives are known. (This would eliminate to need to synthesize the characteristics of two different ships, as the author did with the OLNA/MARINER modification).

C. The results of any such theorectical investigation should be confirmed by model tests.

D. The method should be extended to examine the effect of including an outside excitation on the system, such as waves, wind, etc.

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Appendix I

COMPUTER UTILIZATION

I. Discussion

II. Flow Chart

Appendix I

COMPUTER UTILIZATION

I. Discussion

Three computer programs have been written as part of this investigation. The three programs, described in Appendices II, III and IV, are used in sequence if the investigation of the stability of a two ship system, such as that studied herein, starts at the level of known information with which this thesis was begun. If more information concerning the interaction of the two ships being studied is available, it may be possible to enter the sequence at a later stage. (This sequence of the programs is illustrated in the overall flow chart, Figure AI-1). Precisely because any possible future use of these programs may commence at a different stage, the three programs were not unified into a single large program. The use of the programs will be summarized in the reverse order from that in which they were used in this work.

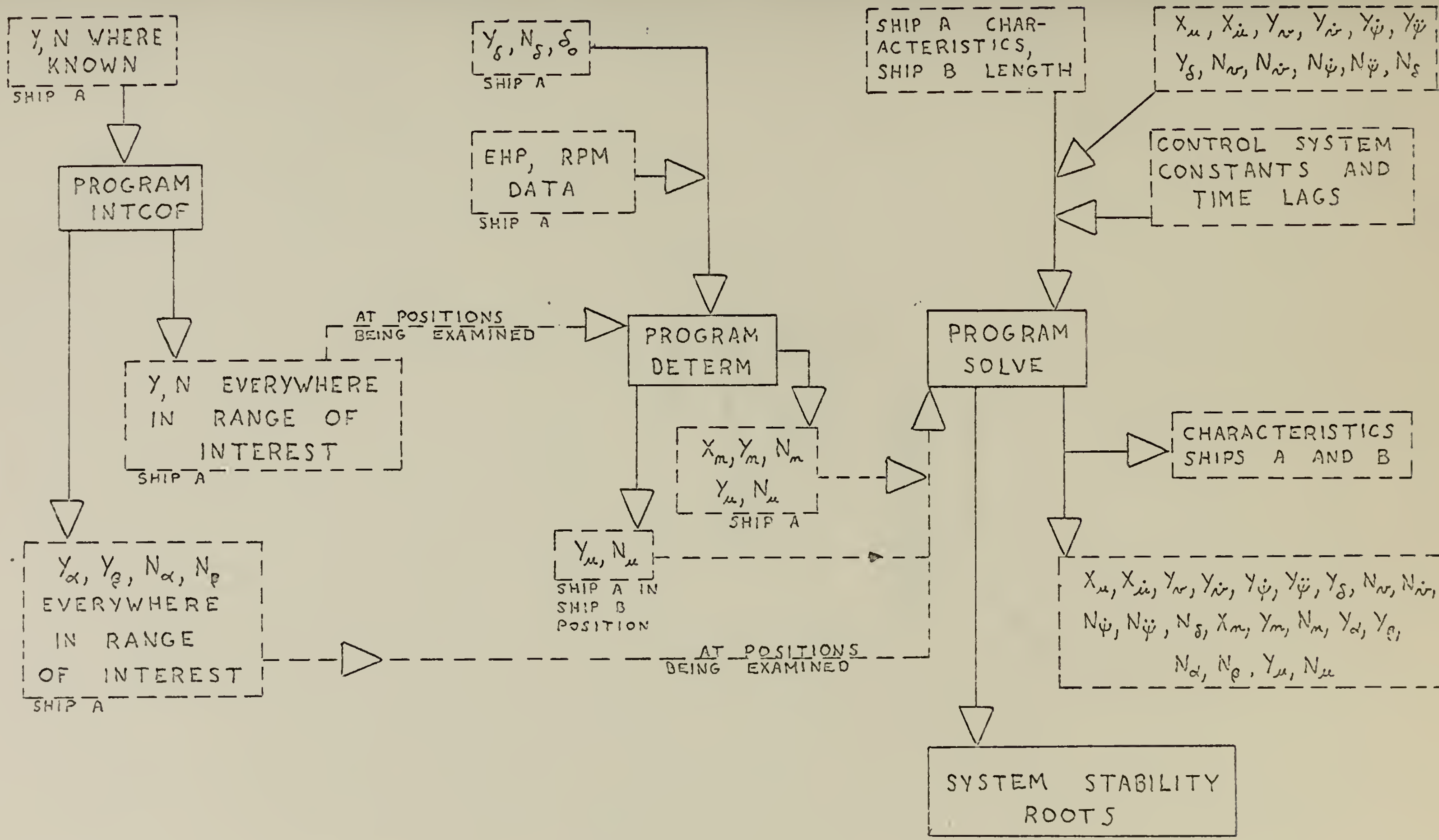
Program SOLVE, the third program used, is the heart of the computer solution and actually solves for the system stability roots. It also requires the greatest amount of known information. If all the interaction and open water hydrodynamic derivatives for Ship A, plus the interaction derivatives for Ship A in Ship B's position, are known, program SOLVE may be used directly.

Program DETERM can be used to get the force and moment derivatives with respect to propeller speed, n, and the

interaction derivatives Y_u and N_u for both ships, if these quantities are not known. To do this, DETERM uses the values of Y and N at the positions of interest, plus Y_δ, N_δ and δ_o open water for Ship A as inputs. EHP and RPM data for Ship A are also required inputs. The output of DETERM then forms part of the required input for SOLVE.

Program INTCOF is used to determine the interaction derivatives of Y and N with respect to α and β, and the values of these quantities at the positions being examined can then be used as part of the SOLVE input. In the process of getting these interaction derivatives throughout the range of interest, program INTCOF also calculates Y and N everywhere in that range, using the interpolation technique described in Chapter III and illustrated in Figure III-1.

The way in which INTCOF and DETERM were written was largely dictated by what information was available concerning the interaction phenomena and were steps toward getting all the derivatives needed for SOLVE. As a result, INTCOF, especially, would hardly be likely to be useful to an investigator starting with a different level of known interaction information. (If, for instance, Y and N were known from model tests everywhere in the range of interest, the interpolating portion of INTCOF would not be needed and the derivatives with respect to α and β could be obtained more directly). Program SOLVE, however, should be directly useful for two ship system stability investigations for any two ships whose hydrodynamic derivatives are all known.



II. FLOW CHART

FIGURE AI-1

Appendix II

PROGRAM INTCOF

I. Discussion

II. Flow Chart

III. Program Listing

IV. Results

Appendix II

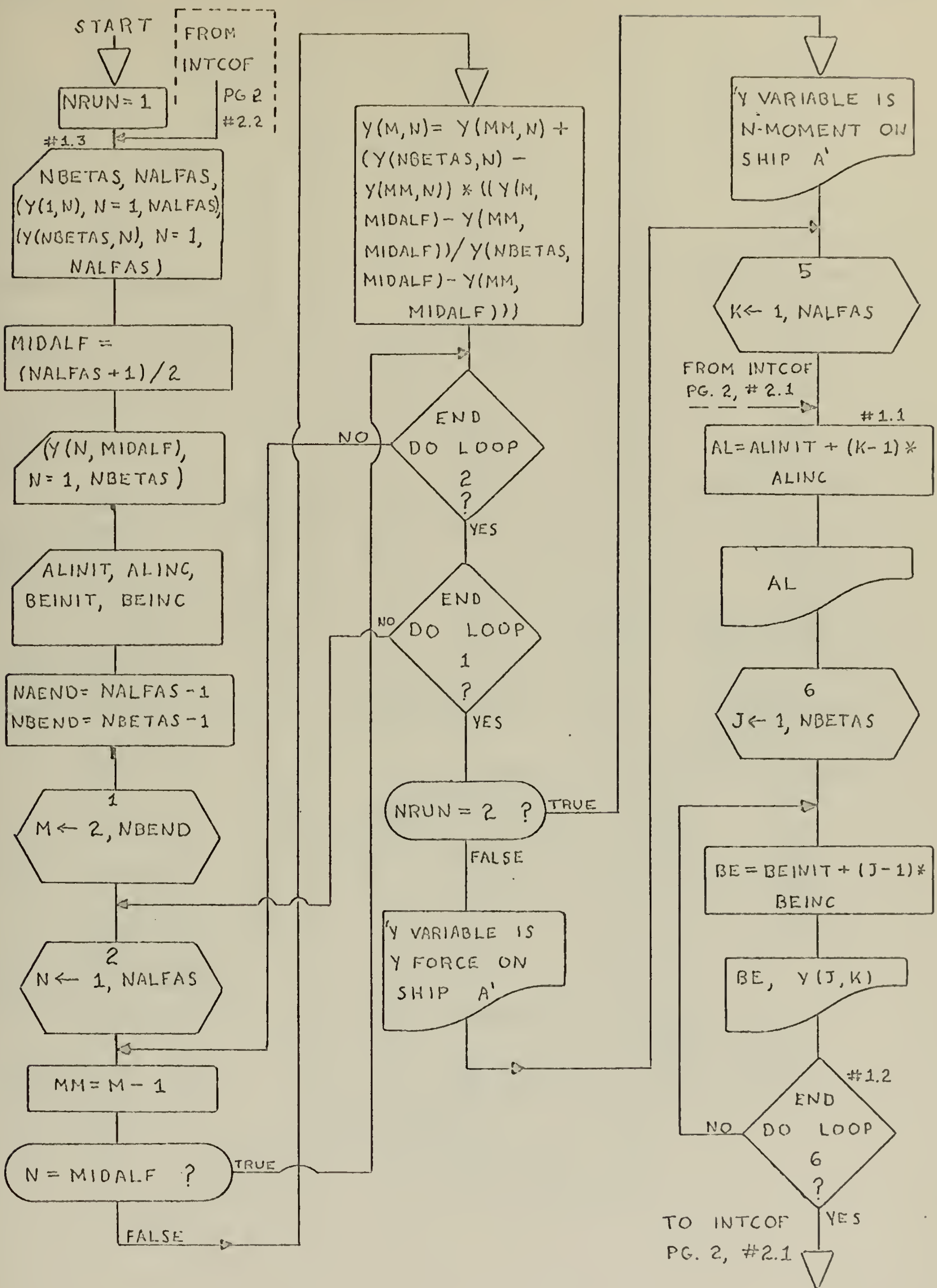
PROGRAM INTCOF

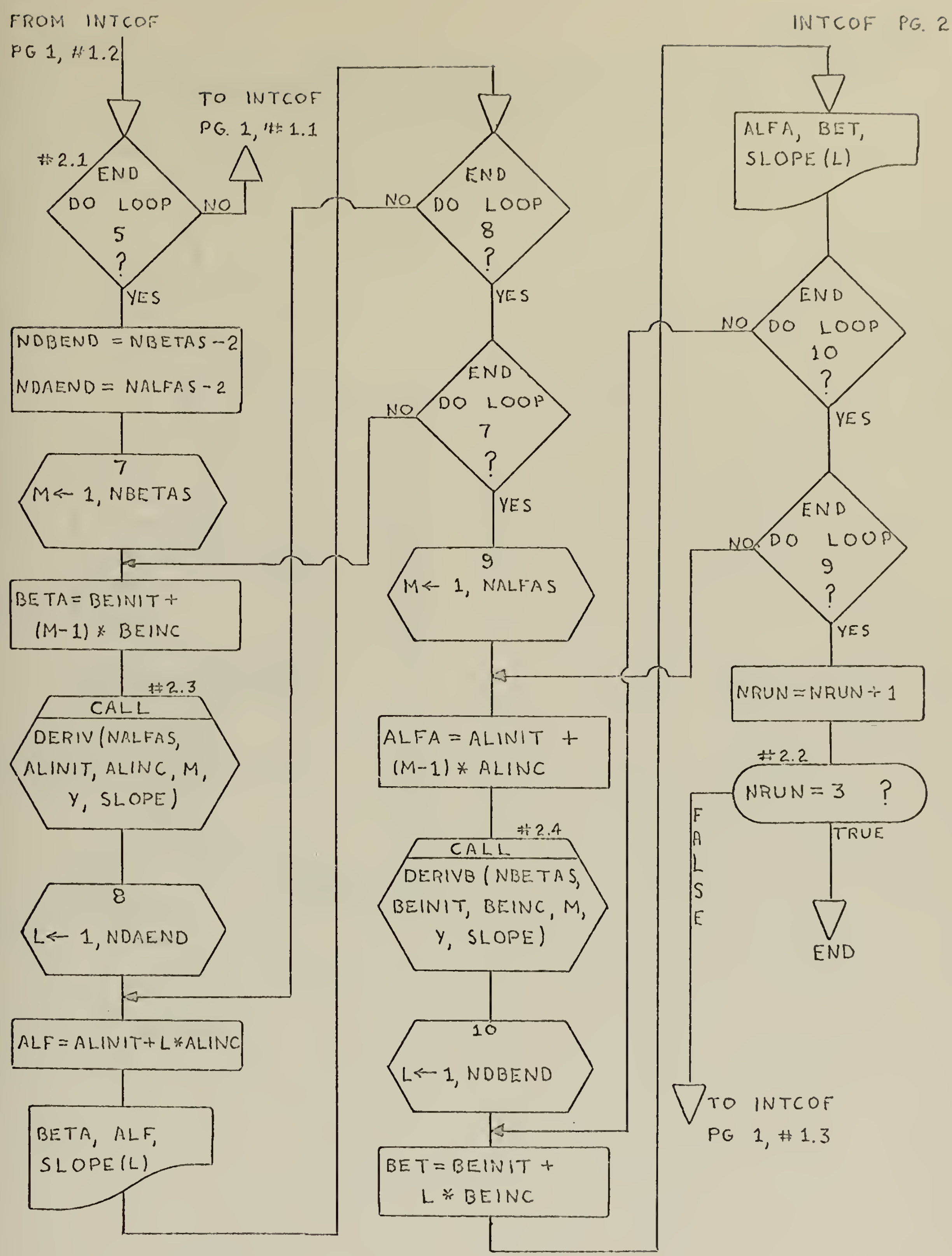
I. Discussion

Program INTCOF takes the values of Y and N where known from reference A-1, interpolates to get Y and N everywhere in the range of interest, then calculates Y_α, Y_β, N_α and N_β everywhere in this range, using subroutines DERIV and DERIVB.

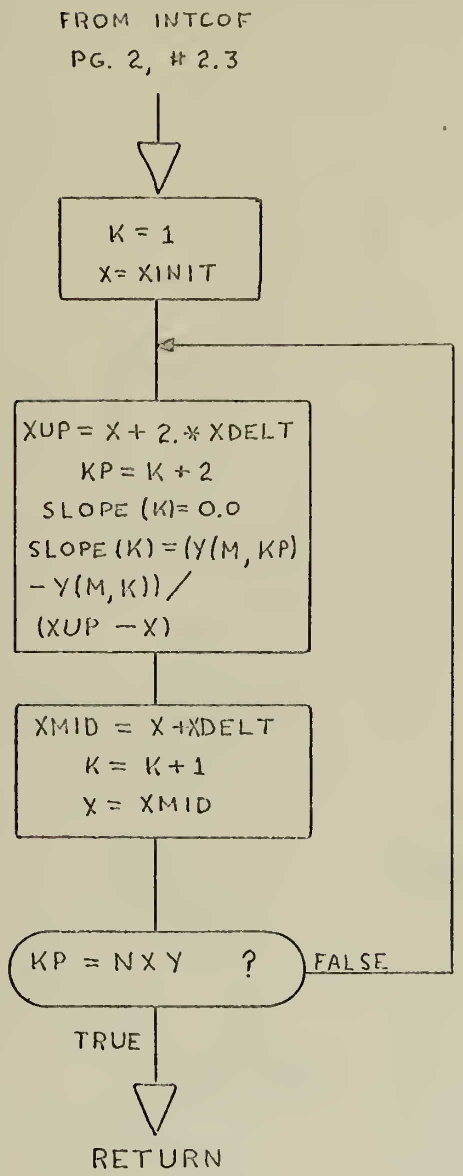
Program INTCOF is highly specialized to the use of the data of reference A-1. Any investigation using Y and N information from another source to get the above four interaction derivatives would be unable to use INTCOF. And, of course, an investigation using the [A-1] data could use the results herein, directly. For these reasons, the input format required for INTCOF will not be described in detail, and the remainder of this Appendix will merely contain a flow chart, program listing and results for INTCOF.

II FLOW CHART — PROGRAM INTCOF

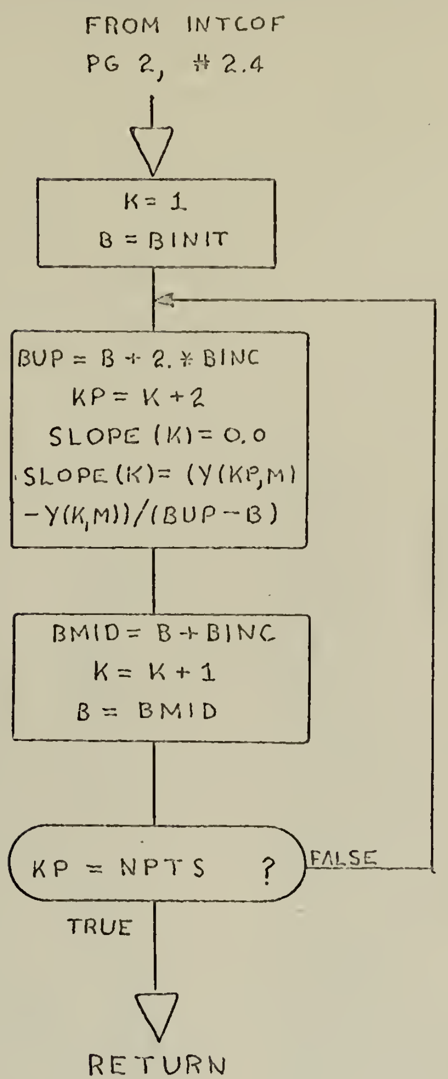



FROM INTCOF PG 1, #1.2
INTCOF PG. 2
TO INTCOF PG. 1, #1.1
#2.1
END DO LOOP 5 ?
NO
YES
NDBEND = NBETAS − 2
NDAEND = NALFAS − 2
7
M ← 1, NBETAS
BETA = BEINIT + (M−1) * BEINC
#2.3
CALL
DERIV (NALFAS, ALINIT, ALINC, M, Y, SLOPE)
8
L ← 1, NDAEND
ALF = ALINIT + L * ALINC
BETA, ALF, SLOPE (L)
END DO LOOP 8 ?
NO
YES
END DO LOOP 7 ?
NO
YES
9
M ← 1, NALFAS
ALFA = ALINIT + (M−1) * ALINC
#2.4
CALL
DERIVB (NBETAS, BEINIT, BEINC, M, Y, SLOPE)
10
L ← 1, NDBEND
BET = BEINIT + L * BEINC
ALFA, BET, SLOPE (L)
END DO LOOP 10 ?
NO
YES
END DO LOOP 9 ?
NO
YES
NRUN = NRUN + 1
#2.2
NRUN = 3 ?
TRUE
FALSE
END
TO INTCOF PG 1, # 1.3

SUBROUTINE DERIV (NXY, XINIT, XDELT, M, Y, SLOPE)



SUBROUTINE DERIVB (NPTS, BINIT, BINC, M, Y, SLOPE)



```
C   III. INTCOF   PROGRAM   LISTING
C
C   PROGRAM ...........INTCOF.....INTERACTION COEFFICIENTS
C   THIS PROGRAM DETERMINES THE VALUES OF INTERACTION
C   FORCES AND MOMENTS EVERYWHERE FROM ALFA=-600 TO +600 FT.
C   AT 50 FOOT INTERVALS, AND SIDE TO SIDE SEPARATION = 50 TO
C   100 FEET, AT 10 FOOT INTERVALS
C   SUBROUTINES ARE THEN CALLED TO DETERMINE DY/D(ALFA),
C   DY/D(BETA), DN/D(ALFA) AND DN/D(BETA) AT THESE LOCATIONS
        DIMENSION Y(10,30),SLOPE(30)
  100 FORMAT(2I4)
  101 FORMAT(5(F10.2))
  102 FORMAT (4F10.2)
  200 FORMAT(///,15X,'Y VARIABLE IS Y FORCE ON SHIP A'/
     115X,'STSD IS SIDE TO SIDE DISTANCE BETWEEN SHIPS A&B')
  201 FORMAT('1',///15X,'Y VARIABLE IS N MOMENT ON SHIP A')
  202 FORMAT(15X,'AT ALFA =',F10.2)
  203 FORMAT(15X,'STSD= ',F10.1,'  Y=',F14.4)
  204 FORMAT(15X,'STSD= ',F8.1,'    ALFA= ',F8.1,'    DY/DA= '
     1,F14.4)
  205 FORMAT(15X,'ALFA= ',F8.1,'    STSD= ' ,F8.1,
     1'    DY/DB=',F14,4)
  206 FORMAT('1',///)
C
C   NRUN IS A COUNTER TO INDICATE THAT THE FIRST TIME THROUGH
C   THE Y VARIABLE IS Y FORCE AND THE SECOND TIME THE Y VAR-
C   IABLE IS N MOMENT.
        NRUN=1
C
C   NBETAS AND NALFAS ARE THE NUMBER OF BETA AND ALFA LO-
C   CATIONS USED.
   12 READ(5,100)NBETAS,NALFAS
C
C   READ KNOWN Y VARIABLES, FIRST IN ALFA THEN IN BETA DIR-
C   ECTION.
        READ (5,101) (Y(1,N),N=1,NALFAS)
```

```
      READ (5,101) (Y(NBETAS,N),N=1,NALFAS)
      MIDALF=(NALFAS+1)/2.
      READ(5,101)(Y(N,MIDALF),N=1,NBETAS)
C
C  READ INITIAL VALUES AND INCREMENTS FOR ALFA AND BETA
      READ (5,102) ALINIT, ALINC, BEINIT, BEINC
      NAEND=NALFAS-1
C
      NBEND=NBETAS-1
C
      DO 1 M=2,NBEND
      DO 2 N=1,NALFAS
      MM=M-1
      IF (N .EQ. MIDALF) GO TO 2
      Y(M,N)=Y(MM,N)+(Y(NBETAS,N)-Y(MM,N))*((Y(M,MIDALF)-Y(M
     1M,MIDALF))/(Y(NBETAS,MIDALF)-Y(MM,MIDALF)))
    2 CONTINUE
    1 CONTINUE
      IF (NRUN .EQ. 2) GO TO 3
      WRITE (6,200)
      GO TO 4
    3 WRITE (6,201)
C
    4 DO 5 K=1,NALFAS
      IF(K- 7)25,26,37
   37 IF (K-13)25,26,38
   38 IF(K-19)25,26,39
   39 IF(K-25)25,26,25
   26 WRITE (6,206)
   25 AL=ALINIT+(K-1)*ALINC
      WRITE (6,202) AL
      DO 6 J=1,NBETAS
      BE=BEINIT+(J-1)*BEINC
      WRITE (6,203) BE,Y(J,K)
    6 CONTINUE
    5 CONTINUE
```

```
      NDBEND = NBETAS-2                                                 INCE0073
      NDAEND = NALFAS-2                                                 INCE0074
      WRITE (6,206)                                                     INCE0075
C                                                                       INCE0076
      DO 7 M=1,NBETAS                                                   INCE0077
      IF(M-3)27,28,40                                                   INCE0078
   40 IF(M-5)27,28,27                                                   INCE0079
   28 WRITE(6,206)                                                      INCE0080
   27 BETA=BEINIT+(M-1)*BEINC                                           INCE0081
      CALL DERIV (NALFAS,ALINIT,ALINC,M,Y,SLOPE)                        INCE0082
      DO 8 L=1,NDAEND                                                   INCE0083
      ALF=ALINIT+L*ALINC                                                INCE0084
      WRITE (6,204) BETA,ALF,SLOPE(L)                                   INCE0085
    8 CONTINUE                                                          INCE0086
    7 CONTINUE                                                          INCE0087
      WRITE(6,206)                                                      INCE0088
C                                                                       INCE0089
      DO 9 M=1,NALFAS                                                   INCE0090
      IF(M-10)29,30,41                                                  INCE0091
   41 IF(M-20)29,30,29                                                  INCE0092
   30 WRITE(6,206)                                                      INCE0093
   29 ALFA=ALINIT+(M-1)*ALINC                                           INCE0094
      CALL DERIVB(NBETAS,BEINIT,BEINC,M,Y,SLOPE)                        INCE0095
      DO 10 L=1,NDBEND                                                  INCE0096
      BET=BEINIT+L*BEINC                                                INCE0097
      WRITE (6,205) ALFA,BET,SLOPE(L)                                   INCE0098
   10 CONTINUE                                                          INCE0099
    9 CONTINUE                                                          INCE0100
C                                                                       INCE0101
      NRUN=NRUN+1                                                       INCE0102
      IF(NRUN .EQ. 3) GO TO 11                                          INCE0103
      GO TO 12                                                          INCE0104
   11 WRITE(6,206)                                                      INCE0105
      STOP                                                              INCE0106
      END                                                               INCE0107
```

```
      SUBROUTINE DERIV (NXY,XINIT,XDELT,M,Y,SLOPE)
C
      DIMENSION Y(10,30),SLOPE(30)
      K=1
      X=XINIT
 1001 XUP=X+2.*XDELT
      KP=K+2
      SLOPE(K)=0.
      SLOPE(K)=(Y(M,KP)-Y(M,K))/(XUP-X)
      XMID=X+XDELT
      K=K+1
      X=XMID
      IF(KP .EQ. NXY) GO TO 1002
      GO TO 1001
 1002 RETURN
      END
```

```
      SUBROUTINE DERIVB (NPTS,BINIT,BINC,M,Y,SLOPE)                     DRVB0001
C                                                                       DRVB0002
      DIMENSION Y(10,30),SLOPE(30)                                      DRVB0003
      K=1                                                               DRVB0004
      B=BINIT                                                           DRVB0005
 1004 BUP=B+2.*BINC                                                     DRVB0006
      KP=K+2                                                            DRVB0007
      SLOPE(K)=0.                                                       DRVB0008
      SLOPE(K)=(Y(KP,M)-Y(K,M))/(BUP-B)                                 DRVB0009
      BMID=B+BINC                                                       DRVB0010
      K=K+1                                                             DRVB0011
      B=BMID                                                            DRVB0012
      IF(KP .EQ. NPTS) GO TO 1003                                       DRVB0013
      GO TO 1004                                                        DRVB0014
 1003 RETURN                                                            DRVB0015
      END                                                               DRVB0016
```

IV. RESULTS

```
Y VARIABLE IS Y FORCE ON SHIP A
STSD IS SIDE TO SIDE DISTANCE BETWEEN SHIPS A&B
AT ALFA =    -600.00
STSD=          50.0   Y=    -0.1400E-03
STSD=          60.0   Y=    -0.1263E-03
STSD=          70.0   Y=    -0.1160E-03
STSD=          80.0   Y=    -0.1058E-03
STSD=          90.0   Y=    -0.9753E-04
STSD=         100.0   Y=    -0.9000E-04
AT ALFA =    -550.00
STSD=          50.0   Y=    -0.2400E-03
STSD=          60.0   Y=    -0.2112E-03
STSD=          70.0   Y=    -0.1897E-03
STSD=          80.0   Y=    -0.1681E-03
STSD=          90.0   Y=    -0.1508E-03
STSD=         100.0   Y=    -0.1350E-03
AT ALFA =    -500.00
STSD=          50.0   Y=    -0.3200E-03
STSD=          60.0   Y=    -0.2762E-03
STSD=          70.0   Y=    -0.2433E-03
STSD=          80.0   Y=    -0.2104E-03
STSD=          90.0   Y=    -0.1841E-03
STSD=         100.0   Y=    -0.1600E-03
AT ALFA =    -450.00
STSD=          50.0   Y=    -0.3800E-03
STSD=          60.0   Y=    -0.3225E-03
STSD=          70.0   Y=    -0.2793E-03
STSD=          80.0   Y=    -0.2362E-03
STSD=          90.0   Y=    -0.2016E-03
STSD=         100.0   Y=    -0.1700E-03
AT ALFA =    -400.00
STSD=          50.0   Y=    -0.4000E-03
STSD=          60.0   Y=    -0.3337E-03
STSD=          70.0   Y=    -0.2840E-03
STSD=          80.0   Y=    -0.2342E-03
STSD=          90.0   Y=    -0.1945E-03
STSD=         100.0   Y=    -0.1580E-03
AT ALFA =    -350.00
STSD=          50.0   Y=    -0.3500E-03
STSD=          60.0   Y=    -0.2870E-03
STSD=          70.0   Y=    -0.2397E-03
STSD=          80.0   Y=    -0.1925E-03
STSD=          90.0   Y=    -0.1547E-03
STSD=         100.0   Y=    -0.1200E-03
```

```
AT ALFA =    -300.00
STSD=        50.0   Y=    -0.2600E-03
STSD=        60.0   Y=    -0.2079E-03
STSD=        70.0   Y=    -0.1689E-03
STSD=        80.0   Y=    -0.1299E-03
STSD=        90.0   Y=    -0.9863E-04
STSD=       100.0   Y=    -0.7000E-04
AT ALFA =    -250.00
STSD=        50.0   Y=    -0.1300E-03
STSD=        60.0   Y=    -0.9334E-04
STSD=        70.0   Y=    -0.6671E-04
STSD=        80.0   Y=    -0.3959E-04
STSD=        90.0   Y=    -0.1739E-04
STSD=       100.0   Y=     0.2000E-05
AT ALFA =    -200.00
STSD=        50.0   Y=     0.4000E-04
STSD=        60.0   Y=     0.5507E-04
STSD=        70.0   Y=     0.6637E-04
STSD=        80.0   Y=     0.7767E-04
STSD=        90.0   Y=     0.8671E-04
STSD=       100.0   Y=     0.9500E-04
AT ALFA =    -150.00
STSD=        50.0   Y=     0.2700E-03
STSD=        60.0   Y=     0.2508E-03
STSD=        70.0   Y=     0.2364E-03
STSD=        80.0   Y=     0.2221E-03
STSD=        90.0   Y=     0.2105E-03
STSD=       100.0   Y=     0.2000E-03
AT ALFA =    -100.00
STSD=        50.0   Y=     0.5000E-03
STSD=        60.0   Y=     0.4507E-03
STSD=        70.0   Y=     0.4137E-03
STSD=        80.0   Y=     0.3767E-03
STSD=        90.0   Y=     0.3471E-03
STSD=       100.0   Y=     0.3200E-03
AT ALFA =     -50.00
STSD=        50.0   Y=     0.7000E-03
STSD=        60.0   Y=     0.6225E-03
STSD=        70.0   Y=     0.5643E-03
STSD=        80.0   Y=     0.5062E-03
STSD=        90.0   Y=     0.4596E-03
STSD=       100.0   Y=     0.4170E-03
```

```
AT ALFA =      0.00
STSD=     50.0   Y=    0.3500E-03
STSD=     60.0   Y=    0.7500E-03
STSD=     70.0   Y=    0.6750E-03
STSD=     80.0   Y=    0.6000E-03
STSD=     90.0   Y=    0.5400E-03
STSD=    100.0   Y=    0.4850E-03
AT ALFA =     50.00
STSD=     50.0   Y=    0.9000E-03
STSD=     60.0   Y=    0.7344E-03
STSD=     70.0   Y=    0.6977E-03
STSD=     80.0   Y=    0.6110E-03
STSD=     90.0   Y=    0.5416E-03
STSD=    100.0   Y=    0.4780E-03
AT ALFA =    100.00
STSD=     50.0   Y=    0.8200E-03
STSD=     60.0   Y=    0.7159E-03
STSD=     70.0   Y=    0.6378E-03
STSD=     80.0   Y=    0.5597E-03
STSD=     90.0   Y=    0.4973E-03
STSD=    100.0   Y=    0.4400E-03
AT ALFA =    150.00
STSD=     50.0   Y=    0.6250E-03
STSD=     60.0   Y=    0.5579E-03
STSD=     70.0   Y=    0.5075E-03
STSD=     80.0   Y=    0.4572E-03
STSD=     90.0   Y=    0.4169E-03
STSD=    100.0   Y=    0.3800E-03
AT ALFA =    200.00
STSD=     50.0   Y=    0.4500E-03
STSD=     60.0   Y=    0.4089E-03
STSD=     70.0   Y=    0.3781E-03
STSD=     80.0   Y=    0.3473E-03
STSD=     90.0   Y=    0.3226E-03
STSD=    100.0   Y=    0.3000E-03
AT ALFA =    250.00
STSD=     50.0   Y=    0.3000E-03
STSD=     60.0   Y=    0.2781E-03
STSD=     70.0   Y=    0.2616E-03
STSD=     80.0   Y=    0.2452E-03
STSD=     90.0   Y=    0.2321E-03
STSD=    100.0   Y=    0.2200E-03
```

```
AT ALFA =     300.00
STSD=        50.0   Y=     0.1700E-03
STSD=        60.0   Y=     0.1673E-03
STSD=        70.0   Y=     0.1652E-03
STSD=        80.0   Y=     0.1632E-03
STSD=        90.0   Y=     0.1615E-03
STSD=       100.0   Y=     0.1600E-03
AT ALFA =     350.00
STSD=        50.0   Y=     0.6000E-04
STSD=        60.0   Y=     0.6822E-04
STSD=        70.0   Y=     0.7438E-04
STSD=        80.0   Y=     0.8055E-04
STSD=        90.0   Y=     0.8548E-04
STSD=       100.0   Y=     0.9000E-04
AT ALFA =     400.00
STSD=        50.0   Y=    -0.2000E-04
STSD=        60.0   Y=    -0.3562E-05
STSD=        70.0   Y=     0.8767E-05
STSD=        80.0   Y=     0.2110E-04
STSD=        90.0   Y=     0.3096E-04
STSD=       100.0   Y=     0.4000E-04
AT ALFA =     450.00
STSD=        50.0   Y=    -0.5000E-04
STSD=        60.0   Y=    -0.3493E-04
STSD=        70.0   Y=    -0.2363E-04
STSD=        80.0   Y=    -0.1233E-04
STSD=        90.0   Y=    -0.3298E-05
STSD=       100.0   Y=     0.5000E-05
AT ALFA =     500.00
STSD=        50.0   Y=    -0.6000E-04
STSD=        60.0   Y=    -0.4904E-04
STSD=        70.0   Y=    -0.4082E-04
STSD=        80.0   Y=    -0.3260E-04
STSD=        90.0   Y=    -0.2603E-04
STSD=       100.0   Y=    -0.2000E-04
AT ALFA =     550.00
STSD=        50.0   Y=    -0.5000E-04
STSD=        60.0   Y=    -0.4699E-04
STSD=        70.0   Y=    -0.4473E-04
STSD=        80.0   Y=    -0.4247E-04
STSD=        90.0   Y=    -0.4065E-04
STSD=       100.0   Y=    -0.3900E-04
```

```
AT ALFA =     600.00
STSD=         50.0   Y=    -0.1000E-04
STSD=         60.0   Y=    -0.2096E-04
STSD=         70.0   Y=    -0.2918E-04
STSD=         80.0   Y=    -0.3740E-04
STSD=         90.0   Y=    -0.4397E-04
STSD=        100.0   Y=    -0.5000E-04
```

STSD=	50.0	ALFA=	-550.0	DY/DA=	-0.1300E-05
STSD=	50.0	ALFA=	-500.0	DY/DA=	-0.1400E-05
STSD=	50.0	ALFA=	-450.0	DY/DA=	-0.3000E-06
STSD=	50.0	ALFA=	-400.0	DY/DA=	0.3000E-06
STSD=	50.0	ALFA=	-350.0	DY/DA=	0.1400E-05
STSD=	50.0	ALFA=	-300.0	DY/DA=	0.2200E-05
STSD=	50.0	ALFA=	-250.0	DY/DA=	0.3000E-05
STSD=	50.0	ALFA=	-200.0	DY/DA=	0.4000E-05
STSD=	50.0	ALFA=	-150.0	DY/DA=	0.4600E-05
STSD=	50.0	ALFA=	-100.0	DY/DA=	0.4300E-05
STSD=	50.0	ALFA=	-50.0	DY/DA=	0.3500E-05
STSD=	50.0	ALFA=	0.0	DY/DA=	0.2000E-05
STSD=	50.0	ALFA=	50.0	DY/DA=	-0.3000E-06
STSD=	50.0	ALFA=	100.0	DY/DA=	-0.2750E-05
STSD=	50.0	ALFA=	150.0	DY/DA=	-0.3700E-05
STSD=	50.0	ALFA=	200.0	DY/DA=	-0.3250E-05
STSD=	50.0	ALFA=	250.0	DY/DA=	-0.2800E-05
STSD=	50.0	ALFA=	300.0	DY/DA=	-0.2400E-05
STSD=	50.0	ALFA=	350.0	DY/DA=	-0.1900E-05
STSD=	50.0	ALFA=	400.0	DY/DA=	-0.1100E-05
STSD=	50.0	ALFA=	450.0	DY/DA=	-0.4000E-06
STSD=	50.0	ALFA=	500.0	DY/DA=	0.0000E 00
STSD=	50.0	ALFA=	550.0	DY/DA=	0.5000E-06
STSD=	60.0	ALFA=	-550.0	DY/DA=	-0.1499E-05
STSD=	60.0	ALFA=	-500.0	DY/DA=	-0.1112E-05
STSD=	60.0	ALFA=	-450.0	DY/DA=	-0.5753E-06
STSD=	60.0	ALFA=	-400.0	DY/DA=	0.3548E-06
STSD=	60.0	ALFA=	-350.0	DY/DA=	0.1258E-05
STSD=	60.0	ALFA=	-300.0	DY/DA=	0.1932E-05
STSD=	60.0	ALFA=	-250.0	DY/DA=	0.2630E-05
STSD=	60.0	ALFA=	-200.0	DY/DA=	0.3447E-05
STSD=	60.0	ALFA=	-150.0	DY/DA=	0.3956E-05
STSD=	60.0	ALFA=	-100.0	DY/DA=	0.3716E-05
STSD=	60.0	ALFA=	-50.0	DY/DA=	0.2993E-05
STSD=	60.0	ALFA=	0.0	DY/DA=	0.1619E-05
STSD=	60.0	ALFA=	50.0	DY/DA=	-0.3411E-06
STSD=	60.0	ALFA=	100.0	DY/DA=	-0.2265E-05
STSD=	60.0	ALFA=	150.0	DY/DA=	-0.3070E-05
STSD=	60.0	ALFA=	200.0	DY/DA=	-0.2798E-05
STSD=	60.0	ALFA=	250.0	DY/DA=	-0.2416E-05
STSD=	60.0	ALFA=	300.0	DY/DA=	-0.2099E-05
STSD=	60.0	ALFA=	350.0	DY/DA=	-0.1708E-05
STSD=	60.0	ALFA=	400.0	DY/DA=	-0.1032E-05
STSD=	60.0	ALFA=	450.0	DY/DA=	-0.4548E-06
STSD=	60.0	ALFA=	500.0	DY/DA=	-0.1205E-06
STSD=	60.0	ALFA=	550.0	DY/DA=	0.2808E-06

STSD=	70.0	ALFA=	-550.0	DY/DA=	-0.1273E-05
STSD=	70.0	ALFA=	-500.0	DY/DA=	-0.3966E-06
STSD=	70.0	ALFA=	-450.0	DY/DA=	-0.4068E-06
STSD=	70.0	ALFA=	-400.0	DY/DA=	0.3959E-06
STSD=	70.0	ALFA=	-350.0	DY/DA=	0.1151E-05
STSD=	70.0	ALFA=	-300.0	DY/DA=	0.1730E-05
STSD=	70.0	ALFA=	-250.0	DY/DA=	0.2353E-05
STSD=	70.0	ALFA=	-200.0	DY/DA=	0.3032E-05
STSD=	70.0	ALFA=	-150.0	DY/DA=	0.3473E-05
STSD=	70.0	ALFA=	-100.0	DY/DA=	0.3279E-05
STSD=	70.0	ALFA=	-50.0	DY/DA=	0.2613E-05
STSD=	70.0	ALFA=	0.0	DY/DA=	0.1334E-05
STSD=	70.0	ALFA=	50.0	DY/DA=	-0.3719E-06
STSD=	70.0	ALFA=	100.0	DY/DA=	-0.1901E-05
STSD=	70.0	ALFA=	150.0	DY/DA=	-0.2597E-05
STSD=	70.0	ALFA=	200.0	DY/DA=	-0.2459E-05
STSD=	70.0	ALFA=	250.0	DY/DA=	-0.2129E-05
STSD=	70.0	ALFA=	300.0	DY/DA=	-0.1873E-05
STSD=	70.0	ALFA=	350.0	DY/DA=	-0.1564E-05
STSD=	70.0	ALFA=	400.0	DY/DA=	-0.9801E-06
STSD=	70.0	ALFA=	450.0	DY/DA=	-0.4959E-06
STSD=	70.0	ALFA=	500.0	DY/DA=	-0.2110E-06
STSD=	70.0	ALFA=	550.0	DY/DA=	0.1164E-06
STSD=	80.0	ALFA=	-550.0	DY/DA=	-0.1047E-05
STSD=	80.0	ALFA=	-500.0	DY/DA=	-0.6808E-06
STSD=	80.0	ALFA=	-450.0	DY/DA=	-0.2384E-06
STSD=	80.0	ALFA=	-400.0	DY/DA=	0.4370E-06
STSD=	80.0	ALFA=	-350.0	DY/DA=	0.1044E-05
STSD=	80.0	ALFA=	-300.0	DY/DA=	0.1529E-05
STSD=	80.0	ALFA=	-250.0	DY/DA=	0.2075E-05
STSD=	80.0	ALFA=	-200.0	DY/DA=	0.2616E-05
STSD=	80.0	ALFA=	-150.0	DY/DA=	0.2990E-05
STSD=	80.0	ALFA=	-100.0	DY/DA=	0.2341E-05
STSD=	80.0	ALFA=	-50.0	DY/DA=	0.2233E-05
STSD=	80.0	ALFA=	0.0	DY/DA=	0.1048E-05
STSD=	80.0	ALFA=	50.0	DY/DA=	-0.4027E-06
STSD=	80.0	ALFA=	100.0	DY/DA=	-0.1538E-05
STSD=	80.0	ALFA=	150.0	DY/DA=	-0.2125E-05
STSD=	80.0	ALFA=	200.0	DY/DA=	-0.2120E-05
STSD=	80.0	ALFA=	250.0	DY/DA=	-0.1841E-05
STSD=	80.0	ALFA=	300.0	DY/DA=	-0.1647E-05
STSD=	80.0	ALFA=	350.0	DY/DA=	-0.1421E-05
STSD=	80.0	ALFA=	400.0	DY/DA=	-0.9233E-06
STSD=	80.0	ALFA=	450.0	DY/DA=	-0.5370E-06
STSD=	80.0	ALFA=	500.0	DY/DA=	-0.3014E-06
STSD=	80.0	ALFA=	550.0	DY/DA=	-0.4795E-07

STSD=	90.0	ALFA=	-550.0	DY/DA=	-0.3653E-06
STSD=	90.0	ALFA=	-500.0	DY/DA=	-0.5082E-06
STSD=	90.0	ALFA=	-450.0	DY/DA=	-0.1036E-06
STSD=	90.0	ALFA=	-400.0	DY/DA=	0.4699E-06
STSD=	90.0	ALFA=	-350.0	DY/DA=	0.9584E-06
STSD=	90.0	ALFA=	-300.0	DY/DA=	0.1368E-05
STSD=	90.0	ALFA=	-250.0	DY/DA=	0.1953E-05
STSD=	90.0	ALFA=	-200.0	DY/DA=	0.2284E-05
STSD=	90.0	ALFA=	-150.0	DY/DA=	0.2604E-05
STSD=	90.0	ALFA=	-100.0	DY/DA=	0.2491E-05
STSD=	90.0	ALFA=	-50.0	DY/DA=	0.1929E-05
STSD=	90.0	ALFA=	0.0	DY/DA=	0.3195E-06
STSD=	90.0	ALFA=	50.0	DY/DA=	-0.4274E-06
STSD=	90.0	ALFA=	100.0	DY/DA=	-0.1247E-05
STSD=	90.0	ALFA=	150.0	DY/DA=	-0.1747E-05
STSD=	90.0	ALFA=	200.0	DY/DA=	-0.1349E-05
STSD=	90.0	ALFA=	250.0	DY/DA=	-0.1511E-05
STSD=	90.0	ALFA=	300.0	DY/DA=	-0.1466E-05
STSD=	90.0	ALFA=	350.0	DY/DA=	-0.1305E-05
STSD=	90.0	ALFA=	400.0	DY/DA=	-0.8377E-06
STSD=	90.0	ALFA=	450.0	DY/DA=	-0.5699E-06
STSD=	90.0	ALFA=	500.0	DY/DA=	-0.3737E-06
STSD=	90.0	ALFA=	550.0	DY/DA=	-0.1795E-06
STSD=	100.0	ALFA=	-550.0	DY/DA=	-0.7000E-06
STSD=	100.0	ALFA=	-500.0	DY/DA=	-0.3500E-06
STSD=	100.0	ALFA=	-450.0	DY/DA=	0.2000E-07
STSD=	100.0	ALFA=	-400.0	DY/DA=	0.5000E-06
STSD=	100.0	ALFA=	-350.0	DY/DA=	0.8300E-06
STSD=	100.0	ALFA=	-300.0	DY/DA=	0.1220E-05
STSD=	100.0	ALFA=	-250.0	DY/DA=	0.1650E-05
STSD=	100.0	ALFA=	-200.0	DY/DA=	0.1980E-05
STSD=	100.0	ALFA=	-150.0	DY/DA=	0.2250E-05
STSD=	100.0	ALFA=	-100.0	DY/DA=	0.2170E-05
STSD=	100.0	ALFA=	-50.0	DY/DA=	0.1550E-05
STSD=	100.0	ALFA=	0.0	DY/DA=	0.6100E-06
STSD=	100.0	ALFA=	50.0	DY/DA=	-0.4500E-06
STSD=	100.0	ALFA=	100.0	DY/DA=	-0.9300E-06
STSD=	100.0	ALFA=	150.0	DY/DA=	-0.1400E-05
STSD=	100.0	ALFA=	200.0	DY/DA=	-0.1600E-05
STSD=	100.0	ALFA=	250.0	DY/DA=	-0.1400E-05
STSD=	100.0	ALFA=	300.0	DY/DA=	-0.1300E-05
STSD=	100.0	ALFA=	350.0	DY/DA=	-0.1200E-05
STSD=	100.0	ALFA=	400.0	DY/DA=	-0.8500E-06
STSD=	100.0	ALFA=	450.0	DY/DA=	-0.6000E-06
STSD=	100.0	ALFA=	500.0	DY/DA=	-0.4400E-06
STSD=	100.0	ALFA=	550.0	DY/DA=	-0.3000E-06

ALFA=	-600.0	STSD=	60.0	DY/DB=	0.1199E-05
ALFA=	-600.0	STSD=	70.0	DY/DB=	0.1027E-05
ALFA=	-600.0	STSD=	80.0	DY/DB=	0.9247E-06
ALFA=	-600.0	STSD=	90.0	DY/DB=	0.7377E-06
ALFA=	-550.0	STSD=	60.0	DY/DB=	0.2517E-05
ALFA=	-550.0	STSD=	70.0	DY/DB=	0.2158E-05
ALFA=	-550.0	STSD=	80.0	DY/DB=	0.1942E-05
ALFA=	-550.0	STSD=	90.0	DY/DB=	0.1654E-05
ALFA=	-500.0	STSD=	60.0	DY/DB=	0.3836E-05
ALFA=	-500.0	STSD=	70.0	DY/DB=	0.3288E-05
ALFA=	-500.0	STSD=	80.0	DY/DB=	0.2959E-05
ALFA=	-500.0	STSD=	90.0	DY/DB=	0.2521E-05
ALFA=	-450.0	STSD=	60.0	DY/DB=	0.5034E-05
ALFA=	-450.0	STSD=	70.0	DY/DB=	0.4315E-05
ALFA=	-450.0	STSD=	80.0	DY/DB=	0.3834E-05
ALFA=	-450.0	STSD=	90.0	DY/DB=	0.3308E-05
ALFA=	-400.0	STSD=	60.0	DY/DB=	0.5301E-05
ALFA=	-400.0	STSD=	70.0	DY/DB=	0.4973E-05
ALFA=	-400.0	STSD=	80.0	DY/DB=	0.4475E-05
ALFA=	-400.0	STSD=	90.0	DY/DB=	0.3812E-05
ALFA=	-350.0	STSD=	60.0	DY/DB=	0.5514E-05
ALFA=	-350.0	STSD=	70.0	DY/DB=	0.4726E-05
ALFA=	-350.0	STSD=	80.0	DY/DB=	0.4253E-05
ALFA=	-350.0	STSD=	90.0	DY/DB=	0.3623E-05
ALFA=	-300.0	STSD=	60.0	DY/DB=	0.4555E-05
ALFA=	-300.0	STSD=	70.0	DY/DB=	0.3904E-05
ALFA=	-300.0	STSD=	80.0	DY/DB=	0.3514E-05
ALFA=	-300.0	STSD=	90.0	DY/DB=	0.2993E-05
ALFA=	-250.0	STSD=	60.0	DY/DB=	0.3164E-05
ALFA=	-250.0	STSD=	70.0	DY/DB=	0.2712E-05
ALFA=	-250.0	STSD=	80.0	DY/DB=	0.2441E-05
ALFA=	-250.0	STSD=	90.0	DY/DB=	0.2079E-05
ALFA=	-200.0	STSD=	60.0	DY/DB=	0.1318E-05
ALFA=	-200.0	STSD=	70.0	DY/DB=	0.1130E-05
ALFA=	-200.0	STSD=	80.0	DY/DB=	0.1017E-05
ALFA=	-200.0	STSD=	90.0	DY/DB=	0.8654E-06

ALFA=	-150.0	STSD=	60.0	DY/DB=	-0.1678E-05
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ALFA=	-150.0	STSD=	80.0	DY/DB=	-0.1295E-05
ALFA=	-150.0	STSD=	90.0	DY/DB=	-0.1103E-05
ALFA=	-100.0	STSD=	60.0	DY/DB=	-0.4315E-05
ALFA=	-100.0	STSD=	70.0	DY/DB=	-0.3609E-05
ALFA=	-100.0	STSD=	80.0	DY/DB=	-0.3329E-05
ALFA=	-100.0	STSD=	90.0	DY/DB=	-0.2836E-05
ALFA=	-50.0	STSD=	60.0	DY/DB=	-0.6734E-05
ALFA=	-50.0	STSD=	70.0	DY/DB=	-0.5815E-05
ALFA=	-50.0	STSD=	80.0	DY/DB=	-0.5234E-05
ALFA=	-50.0	STSD=	90.0	DY/DB=	-0.4458E-05
ALFA=	0.0	STSD=	60.0	DY/DB=	-0.8750E-05
ALFA=	0.0	STSD=	70.0	DY/DB=	-0.7500E-05
ALFA=	0.0	STSD=	80.0	DY/DB=	-0.6750E-05
ALFA=	0.0	STSD=	90.0	DY/DB=	-0.5750E-05
ALFA=	50.0	STSD=	60.0	DY/DB=	-0.1012E-04
ALFA=	50.0	STSD=	70.0	DY/DB=	-0.8671E-05
ALFA=	50.0	STSD=	80.0	DY/DB=	-0.7304E-05
ALFA=	50.0	STSD=	90.0	DY/DB=	-0.6643E-05
ALFA=	100.0	STSD=	60.0	DY/DB=	-0.9110E-05
ALFA=	100.0	STSD=	70.0	DY/DB=	-0.7308E-05
ALFA=	100.0	STSD=	80.0	DY/DB=	-0.7027E-05
ALFA=	100.0	STSD=	90.0	DY/DB=	-0.5936E-05
ALFA=	150.0	STSD=	60.0	DY/DB=	-0.5373E-05
ALFA=	150.0	STSD=	70.0	DY/DB=	-0.5034E-05
ALFA=	150.0	STSD=	80.0	DY/DB=	-0.4531E-05
ALFA=	150.0	STSD=	90.0	DY/DB=	-0.3860E-05
ALFA=	200.0	STSD=	60.0	DY/DB=	-0.3596E-05
ALFA=	200.0	STSD=	70.0	DY/DB=	-0.3082E-05
ALFA=	200.0	STSD=	80.0	DY/DB=	-0.2774E-05
ALFA=	200.0	STSD=	90.0	DY/DB=	-0.2363E-05
ALFA=	250.0	STSD=	60.0	DY/DB=	-0.1918E-05
ALFA=	250.0	STSD=	70.0	DY/DB=	-0.1644E-05
ALFA=	250.0	STSD=	80.0	DY/DB=	-0.1479E-05
ALFA=	250.0	STSD=	90.0	DY/DB=	-0.1260E-05
ALFA=	300.0	STSD=	60.0	DY/DB=	-0.2307E-06
ALFA=	300.0	STSD=	70.0	DY/DB=	-0.2055E-06
ALFA=	300.0	STSD=	80.0	DY/DB=	-0.1849E-06
ALFA=	300.0	STSD=	90.0	DY/DB=	-0.1575E-06

ALFA=	350.0	STSD=	60.0	DY/DB=	0.7192E-06
ALFA=	350.0	STSD=	70.0	DY/DB=	0.6164E-06
ALFA=	350.0	STSD=	80.0	DY/DB=	0.5543E-06
ALFA=	350.0	STSD=	90.0	DY/DB=	0.4726E-06
ALFA=	400.0	STSD=	60.0	DY/DB=	0.1433E-05
ALFA=	400.0	STSD=	70.0	DY/DB=	0.1233E-05
ALFA=	400.0	STSD=	80.0	DY/DB=	0.1110E-05
ALFA=	400.0	STSD=	90.0	DY/DB=	0.9452E-06
ALFA=	450.0	STSD=	60.0	DY/DB=	0.1318E-05
ALFA=	450.0	STSD=	70.0	DY/DB=	0.1130E-05
ALFA=	450.0	STSD=	80.0	DY/DB=	0.1017E-05
ALFA=	450.0	STSD=	90.0	DY/DB=	0.8664E-06
ALFA=	500.0	STSD=	60.0	DY/DB=	0.9539E-06
ALFA=	500.0	STSD=	70.0	DY/DB=	0.8219E-06
ALFA=	500.0	STSD=	80.0	DY/DB=	0.7397E-06
ALFA=	500.0	STSD=	90.0	DY/DB=	0.6301E-06
ALFA=	550.0	STSD=	60.0	DY/DB=	0.2637E-06
ALFA=	550.0	STSD=	70.0	DY/DB=	0.2260E-06
ALFA=	550.0	STSD=	80.0	DY/DB=	0.2034E-06
ALFA=	550.0	STSD=	90.0	DY/DB=	0.1733E-06
ALFA=	600.0	STSD=	60.0	DY/DB=	-0.9589E-06
ALFA=	600.0	STSD=	70.0	DY/DB=	-0.8219E-06
ALFA=	600.0	STSD=	80.0	DY/DB=	-0.7397E-06
ALFA=	600.0	STSD=	90.0	DY/DB=	-0.6301E-06

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Y VARIABLE IS N MOMENT ON SHIP A
AT ALFA =    -600.00
STSD=         50.0   Y=     0.6000E-04
STSD=         60.0   Y=     0.6000E-04
STSD=         70.0   Y=     0.6000E-04
STSD=         80.0   Y=     0.6000E-04
STSD=         90.0   Y=     0.6000E-04
STSD=        100.0   Y=     0.6000E-04
AT ALFA =    -550.00
STSD=         50.0   Y=     0.1100E-03
STSD=         60.0   Y=     0.1036E-03
STSD=         70.0   Y=     0.9786E-04
STSD=         80.0   Y=     0.9280E-04
STSD=         90.0   Y=     0.8890E-04
STSD=        100.0   Y=     0.8500E-04
AT ALFA =    -500.00
STSD=         50.0   Y=     0.1390E-03
STSD=         60.0   Y=     0.1278E-03
STSD=         70.0   Y=     0.1176E-03
STSD=         80.0   Y=     0.1087E-03
STSD=         90.0   Y=     0.1019E-03
STSD=        100.0   Y=     0.9500E-04
AT ALFA =    -450.00
STSD=         50.0   Y=     0.1530E-03
STSD=         60.0   Y=     0.1349E-03
STSD=         70.0   Y=     0.1135E-03
STSD=         80.0   Y=     0.1042E-03
STSD=         90.0   Y=     0.9308E-04
STSD=        100.0   Y=     0.8200E-04
AT ALFA =    -400.00
STSD=         50.0   Y=     0.1400E-03
STSD=         60.0   Y=     0.1197E-03
STSD=         70.0   Y=     0.1012E-03
STSD=         80.0   Y=     0.8497E-04
STSD=         90.0   Y=     0.7249E-04
STSD=        100.0   Y=     0.6000E-04
AT ALFA =    -350.00
STSD=         50.0   Y=     0.7000E-04
STSD=         60.0   Y=     0.5728E-04
STSD=         70.0   Y=     0.4572E-04
STSD=         80.0   Y=     0.3561E-04
STSD=         90.0   Y=     0.2780E-04
STSD=        100.0   Y=     0.2000E-04
```

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AT ALFA =    -300.00
STSD=         50.0   Y=    -0.5000E-04
STSD=         60.0   Y=    -0.4618E-04
STSD=         70.0   Y=    -0.4272E-04
STSD=         80.0   Y=    -0.3968E-04
STSD=         90.0   Y=    -0.3734E-04
STSD=        100.0   Y=    -0.3500E-04
AT ALFA =    -250.00
STSD=         50.0   Y=    -0.1750E-03
STSD=         60.0   Y=    -0.1536E-03
STSD=         70.0   Y=    -0.1342E-03
STSD=         80.0   Y=    -0.1172E-03
STSD=         90.0   Y=    -0.1041E-03
STSD=        100.0   Y=    -0.9100E-04
AT ALFA =    -200.00
STSD=         50.0   Y=    -0.3000E-03
STSD=         60.0   Y=    -0.2598E-03
STSD=         70.0   Y=    -0.2233E-03
STSD=         80.0   Y=    -0.1913E-03
STSD=         90.0   Y=    -0.1667E-03
STSD=        100.0   Y=    -0.1420E-03
AT ALFA =    -150.00
STSD=         50.0   Y=    -0.4100E-03
STSD=         60.0   Y=    -0.3528E-03
STSD=         70.0   Y=    -0.3008E-03
STSD=         80.0   Y=    -0.2552E-03
STSD=         90.0   Y=    -0.2201E-03
STSD=        100.0   Y=    -0.1850E-03
AT ALFA =    -100.00
STSD=         50.0   Y=    -0.4420E-03
STSD=         60.0   Y=    -0.3830E-03
STSD=         70.0   Y=    -0.3294E-03
STSD=         80.0   Y=    -0.2824E-03
STSD=         90.0   Y=    -0.2462E-03
STSD=        100.0   Y=    -0.2100E-03
AT ALFA =     -50.00
STSD=         50.0   Y=    -0.4350E-03
STSD=         60.0   Y=    -0.3785E-03
STSD=         70.0   Y=    -0.3272E-03
STSD=         80.0   Y=    -0.2823E-03
STSD=         90.0   Y=    -0.2476E-03
STSD=        100.0   Y=    -0.2130E-03
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AT ALFA =        0.00
STSD=           50.0   Y=    -0.3740E-03
STSD=           60.0   Y=    -0.3300E-03
STSD=           70.0   Y=    -0.2900E-03
STSD=           80.0   Y=    -0.2550E-03
STSD=           90.0   Y=    -0.2280E-03
STSD=          100.0   Y=    -0.2010E-03
AT ALFA =       50.00
STSD=           50.0   Y=    -0.2800E-03
STSD=           60.0   Y=    -0.2520E-03
STSD=           70.0   Y=    -0.2265E-03
STSD=           80.0   Y=    -0.2043E-03
STSD=           90.0   Y=    -0.1872E-03
STSD=          100.0   Y=    -0.1700E-03
AT ALFA =      100.00
STSD=           50.0   Y=    -0.1500E-03
STSD=           60.0   Y=    -0.1373E-03
STSD=           70.0   Y=    -0.1257E-03
STSD=           80.0   Y=    -0.1156E-03
STSD=           90.0   Y=    -0.1078E-03
STSD=          100.0   Y=    -0.1000E-03
AT ALFA =      150.00
STSD=           50.0   Y=    -0.4500E-04
STSD=           60.0   Y=    -0.4373E-04
STSD=           70.0   Y=    -0.4257E-04
STSD=           80.0   Y=    -0.4156E-04
STSD=           90.0   Y=    -0.4078E-04
STSD=          100.0   Y=    -0.4000E-04
AT ALFA =      200.00
STSD=           50.0   Y=     0.4200E-04
STSD=           60.0   Y=     0.3640E-04
STSD=           70.0   Y=     0.3132E-04
STSD=           80.0   Y=     0.2687E-04
STSD=           90.0   Y=     0.2343E-04
STSD=          100.0   Y=     0.2000E-04
AT ALFA =      250.00
STSD=           50.0   Y=     0.1100E-03
STSD=           60.0   Y=     0.9728E-04
STSD=           70.0   Y=     0.8572E-04
STSD=           80.0   Y=     0.7561E-04
STSD=           90.0   Y=     0.6780E-04
STSD=          100.0   Y=     0.6000E-04
```

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AT ALFA =     300.00
STSD=       50.0   Y=     0.1400E-03
STSD=       60.0   Y=     0.1247E-03
STSD=       70.0   Y=     0.1109E-03
STSD=       80.0   Y=     0.9873E-04
STSD=       90.0   Y=     0.8936E-04
STSD=      100.0   Y=     0.8000E-04
AT ALFA =     350.00
STSD=       50.0   Y=     0.1320E-03
STSD=       60.0   Y=     0.1193E-03
STSD=       70.0   Y=     0.1077E-03
STSD=       80.0   Y=     0.9761E-04
STSD=       90.0   Y=     0.8980E-04
STSD=      100.0   Y=     0.8200E-04
AT ALFA =     400.00
STSD=       50.0   Y=     0.1200E-03
STSD=       60.0   Y=     0.1060E-03
STSD=       70.0   Y=     0.9329E-04
STSD=       80.0   Y=     0.8217E-04
STSD=       90.0   Y=     0.7358E-04
STSD=      100.0   Y=     0.6500E-04
AT ALFA =     450.00
STSD=       50.0   Y=     0.1020E-03
STSD=       60.0   Y=     0.8750E-04
STSD=       70.0   Y=     0.7432E-04
STSD=       80.0   Y=     0.6279E-04
STSD=       90.0   Y=     0.5390E-04
STSD=      100.0   Y=     0.4500E-04
AT ALFA =     500.00
STSD=       50.0   Y=     0.7500E-04
STSD=       60.0   Y=     0.6355E-04
STSD=       70.0   Y=     0.5315E-04
STSD=       80.0   Y=     0.4405E-04
STSD=       90.0   Y=     0.3702E-04
STSD=      100.0   Y=     0.3000E-04
AT ALFA =     550.00
STSD=       50.0   Y=     0.4500E-04
STSD=       60.0   Y=     0.3864E-04
STSD=       70.0   Y=     0.3286E-04
STSD=       80.0   Y=     0.2760E-04
STSD=       90.0   Y=     0.2390E-04
STSD=      100.0   Y=     0.2000E-04
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AT ALFA = 600.00

STSD=	50.0	Y=	0.9000E-05
STSD=	60.0	Y=	0.1053E-04
STSD=	70.0	Y=	0.1191E-04
STSD=	80.0	Y=	0.1313E-04
STSD=	90.0	Y=	0.1406E-04
STSD=	100.0	Y=	0.1500E-04

STSD=	50.0	ALFA=	-550.0	DY/DA=	0.7900E-06
STSD=	50.0	ALFA=	-500.0	DY/DA=	0.4300E-06
STSD=	50.0	ALFA=	-450.0	DY/DA=	0.1000E-07
STSD=	50.0	ALFA=	-400.0	DY/DA=	-0.8300E-06
STSD=	50.0	ALFA=	-350.0	DY/DA=	-0.1900E-05
STSD=	50.0	ALFA=	-300.0	DY/DA=	-0.2450E-05
STSD=	50.0	ALFA=	-250.0	DY/DA=	-0.2500E-05
STSD=	50.0	ALFA=	-200.0	DY/DA=	-0.2350E-05
STSD=	50.0	ALFA=	-150.0	DY/DA=	-0.1420E-05
STSD=	50.0	ALFA=	-100.0	DY/DA=	-0.2500E-06
STSD=	50.0	ALFA=	-50.0	DY/DA=	0.6300E-06
STSD=	50.0	ALFA=	0.0	DY/DA=	0.1550E-05
STSD=	50.0	ALFA=	50.0	DY/DA=	0.2240E-05
STSD=	50.0	ALFA=	100.0	DY/DA=	0.2350E-05
STSD=	50.0	ALFA=	150.0	DY/DA=	0.1920E-05
STSD=	50.0	ALFA=	200.0	DY/DA=	0.1550E-05
STSD=	50.0	ALFA=	250.0	DY/DA=	0.9300E-06
STSD=	50.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	50.0	ALFA=	350.0	DY/DA=	-0.2000E-06
STSD=	50.0	ALFA=	400.0	DY/DA=	-0.3000E-06
STSD=	50.0	ALFA=	450.0	DY/DA=	-0.4500E-06
STSD=	50.0	ALFA=	500.0	DY/DA=	-0.5700E-06
STSD=	50.0	ALFA=	550.0	DY/DA=	-0.6600E-06
STSD=	60.0	ALFA=	-550.0	DY/DA=	0.6781E-06
STSD=	60.0	ALFA=	-500.0	DY/DA=	0.3130E-06
STSD=	60.0	ALFA=	-450.0	DY/DA=	-0.8156E-07
STSD=	60.0	ALFA=	-400.0	DY/DA=	-0.7766E-06
STSD=	60.0	ALFA=	-350.0	DY/DA=	-0.1553E-05
STSD=	60.0	ALFA=	-300.0	DY/DA=	-0.2109E-05
STSD=	60.0	ALFA=	-250.0	DY/DA=	-0.2136E-05
STSD=	60.0	ALFA=	-200.0	DY/DA=	-0.1991E-05
STSD=	60.0	ALFA=	-150.0	DY/DA=	-0.1232E-05
STSD=	60.0	ALFA=	-100.0	DY/DA=	-0.2576E-06
STSD=	60.0	ALFA=	-50.0	DY/DA=	0.5299E-06
STSD=	60.0	ALFA=	0.0	DY/DA=	0.1265E-05
STSD=	60.0	ALFA=	50.0	DY/DA=	0.1927E-05
STSD=	60.0	ALFA=	100.0	DY/DA=	0.2083E-05
STSD=	60.0	ALFA=	150.0	DY/DA=	0.1737E-05
STSD=	60.0	ALFA=	200.0	DY/DA=	0.1410E-05
STSD=	60.0	ALFA=	250.0	DY/DA=	0.8834E-06
STSD=	60.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	60.0	ALFA=	350.0	DY/DA=	-0.1873E-06
STSD=	60.0	ALFA=	400.0	DY/DA=	-0.3178E-06
STSD=	60.0	ALFA=	450.0	DY/DA=	-0.4246E-06
STSD=	60.0	ALFA=	500.0	DY/DA=	-0.4866E-06
STSD=	60.0	ALFA=	550.0	DY/DA=	-0.5503E-06

STSD=	70.0	ALFA=	-550.0	DY/DA=	0.5764E-06
STSD=	70.0	ALFA=	-500.0	DY/DA=	0.2066E-06
STSD=	70.0	ALFA=	-450.0	DY/DA=	-0.1648E-06
STSD=	70.0	ALFA=	-400.0	DY/DA=	-0.7280E-06
STSD=	70.0	ALFA=	-350.0	DY/DA=	-0.1439E-05
STSD=	70.0	ALFA=	-300.0	DY/DA=	-0.1799E-05
STSD=	70.0	ALFA=	-250.0	DY/DA=	-0.1906E-05
STSD=	70.0	ALFA=	-200.0	DY/DA=	-0.1665E-05
STSD=	70.0	ALFA=	-150.0	DY/DA=	-0.1061E-05
STSD=	70.0	ALFA=	-100.0	DY/DA=	-0.2646E-06
STSD=	70.0	ALFA=	-50.0	DY/DA=	0.3935E-06
STSD=	70.0	ALFA=	0.0	DY/DA=	0.1006E-05
STSD=	70.0	ALFA=	50.0	DY/DA=	0.1643E-05
STSD=	70.0	ALFA=	100.0	DY/DA=	0.1340E-05
STSD=	70.0	ALFA=	150.0	DY/DA=	0.1570E-05
STSD=	70.0	ALFA=	200.0	DY/DA=	0.1283E-05
STSD=	70.0	ALFA=	250.0	DY/DA=	0.7955E-06
STSD=	70.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	70.0	ALFA=	350.0	DY/DA=	-0.1757E-06
STSD=	70.0	ALFA=	400.0	DY/DA=	-0.3340E-06
STSD=	70.0	ALFA=	450.0	DY/DA=	-0.4014E-06
STSD=	70.0	ALFA=	500.0	DY/DA=	-0.4146E-06
STSD=	70.0	ALFA=	550.0	DY/DA=	-0.4124E-06
STSD=	80.0	ALFA=	-550.0	DY/DA=	0.4373E-06
STSD=	80.0	ALFA=	-500.0	DY/DA=	0.1136E-06
STSD=	80.0	ALFA=	-450.0	DY/DA=	-0.2376E-06
STSD=	80.0	ALFA=	-400.0	DY/DA=	-0.6355E-06
STSD=	80.0	ALFA=	-350.0	DY/DA=	-0.1247E-05
STSD=	80.0	ALFA=	-300.0	DY/DA=	-0.1528E-05
STSD=	80.0	ALFA=	-250.0	DY/DA=	-0.1516E-05
STSD=	80.0	ALFA=	-200.0	DY/DA=	-0.1380E-05
STSD=	80.0	ALFA=	-150.0	DY/DA=	-0.9110E-06
STSD=	80.0	ALFA=	-100.0	DY/DA=	-0.2706E-06
STSD=	80.0	ALFA=	-50.0	DY/DA=	0.2742E-06
STSD=	80.0	ALFA=	0.0	DY/DA=	0.7796E-06
STSD=	80.0	ALFA=	50.0	DY/DA=	0.1394E-05
STSD=	80.0	ALFA=	100.0	DY/DA=	0.1628E-05
STSD=	80.0	ALFA=	150.0	DY/DA=	0.1425E-05
STSD=	80.0	ALFA=	200.0	DY/DA=	0.1172E-05
STSD=	80.0	ALFA=	250.0	DY/DA=	0.7186E-06
STSD=	80.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	80.0	ALFA=	350.0	DY/DA=	-0.1656E-06
STSD=	80.0	ALFA=	400.0	DY/DA=	-0.3482E-06
STSD=	80.0	ALFA=	450.0	DY/DA=	-0.3312E-06
STSD=	80.0	ALFA=	500.0	DY/DA=	-0.3499E-06
STSD=	80.0	ALFA=	550.0	DY/DA=	-0.3092E-06

STSD=	90.0	ALFA=	-550.0	DY/DA=	0.4187E-06
STSD=	90.0	ALFA=	-500.0	DY/DA=	0.4179E-07
STSD=	90.0	ALFA=	-450.0	DY/DA=	-0.2958E-06
STSD=	90.0	ALFA=	-400.0	DY/DA=	-0.6528E-06
STSD=	90.0	ALFA=	-350.0	DY/DA=	-0.1098E-05
STSD=	90.0	ALFA=	-300.0	DY/DA=	-0.1319E-05
STSD=	90.0	ALFA=	-250.0	DY/DA=	-0.1293E-05
STSD=	90.0	ALFA=	-200.0	DY/DA=	-0.1160E-05
STSD=	90.0	ALFA=	-150.0	DY/DA=	-0.7955E-06
STSD=	90.0	ALFA=	-100.0	DY/DA=	-0.2753E-06
STSD=	90.0	ALFA=	-50.0	DY/DA=	0.1821E-06
STSD=	90.0	ALFA=	0.0	DY/DA=	0.6048E-06
STSD=	90.0	ALFA=	50.0	DY/DA=	0.1202E-05
STSD=	90.0	ALFA=	100.0	DY/DA=	0.1464E-05
STSD=	90.0	ALFA=	150.0	DY/DA=	0.1312E-05
STSD=	90.0	ALFA=	200.0	DY/DA=	0.1085E-05
STSD=	90.0	ALFA=	250.0	DY/DA=	0.5593E-06
STSD=	90.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	90.0	ALFA=	350.0	DY/DA=	-0.1578E-06
STSD=	90.0	ALFA=	400.0	DY/DA=	-0.3591E-06
STSD=	90.0	ALFA=	450.0	DY/DA=	-0.3656E-06
STSD=	90.0	ALFA=	500.0	DY/DA=	-0.2999E-06
STSD=	90.0	ALFA=	550.0	DY/DA=	-0.2296E-06
STSD=	100.0	ALFA=	-550.0	DY/DA=	0.3500E-06
STSD=	100.0	ALFA=	-500.0	DY/DA=	-0.3000E-07
STSD=	100.0	ALFA=	-450.0	DY/DA=	-0.3500E-06
STSD=	100.0	ALFA=	-400.0	DY/DA=	-0.6200E-06
STSD=	100.0	ALFA=	-350.0	DY/DA=	-0.9500E-06
STSD=	100.0	ALFA=	-300.0	DY/DA=	-0.1110E-05
STSD=	100.0	ALFA=	-250.0	DY/DA=	-0.1070E-05
STSD=	100.0	ALFA=	-200.0	DY/DA=	-0.9400E-06
STSD=	100.0	ALFA=	-150.0	DY/DA=	-0.6800E-06
STSD=	100.0	ALFA=	-100.0	DY/DA=	-0.2300E-06
STSD=	100.0	ALFA=	-50.0	DY/DA=	0.9000E-07
STSD=	100.0	ALFA=	0.0	DY/DA=	0.4300E-06
STSD=	100.0	ALFA=	50.0	DY/DA=	0.1010E-05
STSD=	100.0	ALFA=	100.0	DY/DA=	0.1300E-05
STSD=	100.0	ALFA=	150.0	DY/DA=	0.1200E-05
STSD=	100.0	ALFA=	200.0	DY/DA=	0.1000E-05
STSD=	100.0	ALFA=	250.0	DY/DA=	0.6000E-06
STSD=	100.0	ALFA=	300.0	DY/DA=	0.2200E-06
STSD=	100.0	ALFA=	350.0	DY/DA=	-0.1500E-06
STSD=	100.0	ALFA=	400.0	DY/DA=	-0.3700E-06
STSD=	100.0	ALFA=	450.0	DY/DA=	-0.3500E-06
STSD=	100.0	ALFA=	500.0	DY/DA=	-0.2500E-06
STSD=	100.0	ALFA=	550.0	DY/DA=	-0.1500E-06

ALFA=	-600.0	STSD=	60.0	DY/DB=	0.0000E 00
ALFA=	-600.0	STSD=	70.0	DY/DB=	0.0000E 00
ALFA=	-600.0	STSD=	80.0	DY/DB=	0.0000E 00
ALFA=	-600.0	STSD=	90.0	DY/DB=	0.0000E 00
ALFA=	-550.0	STSD=	60.0	DY/DB=	-0.6069E-06
ALFA=	-550.0	STSD=	70.0	DY/DB=	-0.5419E-06
ALFA=	-550.0	STSD=	80.0	DY/DB=	-0.4430E-06
ALFA=	-550.0	STSD=	90.0	DY/DB=	-0.3902E-06
ALFA=	-500.0	STSD=	60.0	DY/DB=	-0.1068E-05
ALFA=	-500.0	STSD=	70.0	DY/DB=	-0.9538E-06
ALFA=	-500.0	STSD=	80.0	DY/DB=	-0.7834E-06
ALFA=	-500.0	STSD=	90.0	DY/DB=	-0.6367E-06
ALFA=	-450.0	STSD=	60.0	DY/DB=	-0.1724E-05
ALFA=	-450.0	STSD=	70.0	DY/DB=	-0.1539E-05
ALFA=	-450.0	STSD=	80.0	DY/DB=	-0.1272E-05
ALFA=	-450.0	STSD=	90.0	DY/DB=	-0.1108E-05
ALFA=	-400.0	STSD=	60.0	DY/DB=	-0.1942E-05
ALFA=	-400.0	STSD=	70.0	DY/DB=	-0.1734E-05
ALFA=	-400.0	STSD=	80.0	DY/DB=	-0.1434E-05
ALFA=	-400.0	STSD=	90.0	DY/DB=	-0.1249E-05
ALFA=	-350.0	STSD=	60.0	DY/DB=	-0.1214E-05
ALFA=	-350.0	STSD=	70.0	DY/DB=	-0.1034E-05
ALFA=	-350.0	STSD=	80.0	DY/DB=	-0.8060E-06
ALFA=	-350.0	STSD=	90.0	DY/DB=	-0.7303E-06
ALFA=	-300.0	STSD=	60.0	DY/DB=	0.3642E-06
ALFA=	-300.0	STSD=	70.0	DY/DB=	0.3251E-06
ALFA=	-300.0	STSD=	80.0	DY/DB=	0.2638E-06
ALFA=	-300.0	STSD=	90.0	DY/DB=	0.2341E-06
ALFA=	-250.0	STSD=	60.0	DY/DB=	0.2039E-05
ALFA=	-250.0	STSD=	70.0	DY/DB=	0.1321E-05
ALFA=	-250.0	STSD=	80.0	DY/DB=	0.1505E-05
ALFA=	-250.0	STSD=	90.0	DY/DB=	0.1311E-05
ALFA=	-200.0	STSD=	60.0	DY/DB=	0.3336E-05
ALFA=	-200.0	STSD=	70.0	DY/DB=	0.3425E-05
ALFA=	-200.0	STSD=	80.0	DY/DB=	0.2331E-05
ALFA=	-200.0	STSD=	90.0	DY/DB=	0.2466E-05

```
ALFA=   -150.0   STSD=   60.0   DY/DB=    0.5462E-05
ALFA=   -150.0   STSD=   70.0   DY/DB=    0.4377E-05
ALFA=   -150.0   STSD=   80.0   DY/DB=    0.4032E-05
ALFA=   -150.0   STSD=   90.0   DY/DB=    0.3512E-05
ALFA=   -100.0   STSD=   60.0   DY/DB=    0.5632E-05
ALFA=   -100.0   STSD=   70.0   DY/DB=    0.5029E-05
ALFA=   -100.0   STSD=   80.0   DY/DB=    0.4157E-05
ALFA=   -100.0   STSD=   90.0   DY/DB=    0.3621E-05
ALFA=    -50.0   STSD=   60.0   DY/DB=    0.5390E-05
ALFA=    -50.0   STSD=   70.0   DY/DB=    0.4812E-05
ALFA=    -50.0   STSD=   80.0   DY/DB=    0.3978E-05
ALFA=    -50.0   STSD=   90.0   DY/DB=    0.3465E-05
ALFA=      0.0   STSD=   60.0   DY/DB=    0.4200E-05
ALFA=      0.0   STSD=   70.0   DY/DB=    0.3750E-05
ALFA=      0.0   STSD=   80.0   DY/DB=    0.3100E-05
ALFA=      0.0   STSD=   90.0   DY/DB=    0.2700E-05
ALFA=     50.0   STSD=   60.0   DY/DB=    0.2671E-05
ALFA=     50.0   STSD=   70.0   DY/DB=    0.2334E-05
ALFA=     50.0   STSD=   80.0   DY/DB=    0.1971E-05
ALFA=     50.0   STSD=   90.0   DY/DB=    0.1717E-05
ALFA=    100.0   STSD=   60.0   DY/DB=    0.1214E-05
ALFA=    100.0   STSD=   70.0   DY/DB=    0.1034E-05
ALFA=    100.0   STSD=   80.0   DY/DB=    0.8960E-06
ALFA=    100.0   STSD=   90.0   DY/DB=    0.7303E-06
ALFA=    150.0   STSD=   60.0   DY/DB=    0.1214E-06
ALFA=    150.0   STSD=   70.0   DY/DB=    0.1034E-06
ALFA=    150.0   STSD=   80.0   DY/DB=    0.8960E-07
ALFA=    150.0   STSD=   90.0   DY/DB=    0.7303E-07
ALFA=    200.0   STSD=   60.0   DY/DB=   -0.5341E-06
ALFA=    200.0   STSD=   70.0   DY/DB=   -0.4759E-06
ALFA=    200.0   STSD=   80.0   DY/DB=   -0.3942E-06
ALFA=    200.0   STSD=   90.0   DY/DB=   -0.3434E-06
ALFA=    250.0   STSD=   60.0   DY/DB=   -0.1214E-05
ALFA=    250.0   STSD=   70.0   DY/DB=   -0.1034E-05
ALFA=    250.0   STSD=   80.0   DY/DB=   -0.8960E-06
ALFA=    250.0   STSD=   90.0   DY/DB=   -0.7303E-06
ALFA=    300.0   STSD=   60.0   DY/DB=   -0.1457E-05
ALFA=    300.0   STSD=   70.0   DY/DB=   -0.1301E-05
ALFA=    300.0   STSD=   80.0   DY/DB=   -0.1075E-05
ALFA=    300.0   STSD=   90.0   DY/DB=   -0.9364E-06
```

ALFA=	350.0	STSD=	60.0	DY/DB=	-0.1214E-05
ALFA=	350.0	STSD=	70.0	DY/DB=	-0.1034E-05
ALFA=	350.0	STSD=	80.0	DY/DB=	-0.8960E-06
ALFA=	350.0	STSD=	90.0	DY/DB=	-0.7803E-06
ALFA=	400.0	STSD=	60.0	DY/DB=	-0.1335E-05
ALFA=	400.0	STSD=	70.0	DY/DB=	-0.1192E-05
ALFA=	400.0	STSD=	80.0	DY/DB=	-0.9356E-06
ALFA=	400.0	STSD=	90.0	DY/DB=	-0.8534E-06
ALFA=	450.0	STSD=	60.0	DY/DB=	-0.1334E-05
ALFA=	450.0	STSD=	70.0	DY/DB=	-0.1236E-05
ALFA=	450.0	STSD=	80.0	DY/DB=	-0.1021E-05
ALFA=	450.0	STSD=	90.0	DY/DB=	-0.8836E-06
ALFA=	500.0	STSD=	60.0	DY/DB=	-0.1002E-05
ALFA=	500.0	STSD=	70.0	DY/DB=	-0.9754E-06
ALFA=	500.0	STSD=	80.0	DY/DB=	-0.8064E-06
ALFA=	500.0	STSD=	90.0	DY/DB=	-0.7023E-06
ALFA=	550.0	STSD=	60.0	DY/DB=	-0.6069E-06
ALFA=	550.0	STSD=	70.0	DY/DB=	-0.5419E-06
ALFA=	550.0	STSD=	80.0	DY/DB=	-0.4430E-06
ALFA=	550.0	STSD=	90.0	DY/DB=	-0.3902E-06
ALFA=	600.0	STSD=	60.0	DY/DB=	0.1457E-06
ALFA=	600.0	STSD=	70.0	DY/DB=	0.1301E-06
ALFA=	600.0	STSD=	80.0	DY/DB=	0.1075E-06
ALFA=	600.0	STSD=	90.0	DY/DB=	0.9364E-07

Appendix III

PROGRAM DETERM

I. Discussion

II. Input/Output Format

III. Flow Chart

IV. Program Listing

V. Sample Results

Appendix III

PROGRAM DETERM

I. Discussion

Program DETERM determines the hydrodynamic derivatives with respect to propeller speed, n, which were not available in the literature surveyed. It also determines the variation of interaction effects with velocity, u. Thus the outputs are X_n, Y_n and N_n for Ship A; Y_u and N_u for Ship A; and Y_u and N_u for Ship A in Ship B's position. These latter values, when used as inputs in program SOLVE are modified to their appropriate values for Ship B in Ship B's position. (X_n, Y_n and N_n are position-independent.)

The values of X_n, Y_n and N_n are calculated by using the known values of X, Y and N exerted by the propeller at equilibrium speed in open water and assuming that propeller forces and moments vary as the square of propeller speed. The derivatives with respect to u also are calculated by taking the known values of Y and N at equilibrium and assuming that they vary as the square of the velocity, u. (The procedure followed in calculating these quantities is described in detail in Chapter III, PORCEDURES.)

II. Input/Output Format

Program DETERM's input is ordered as follows:

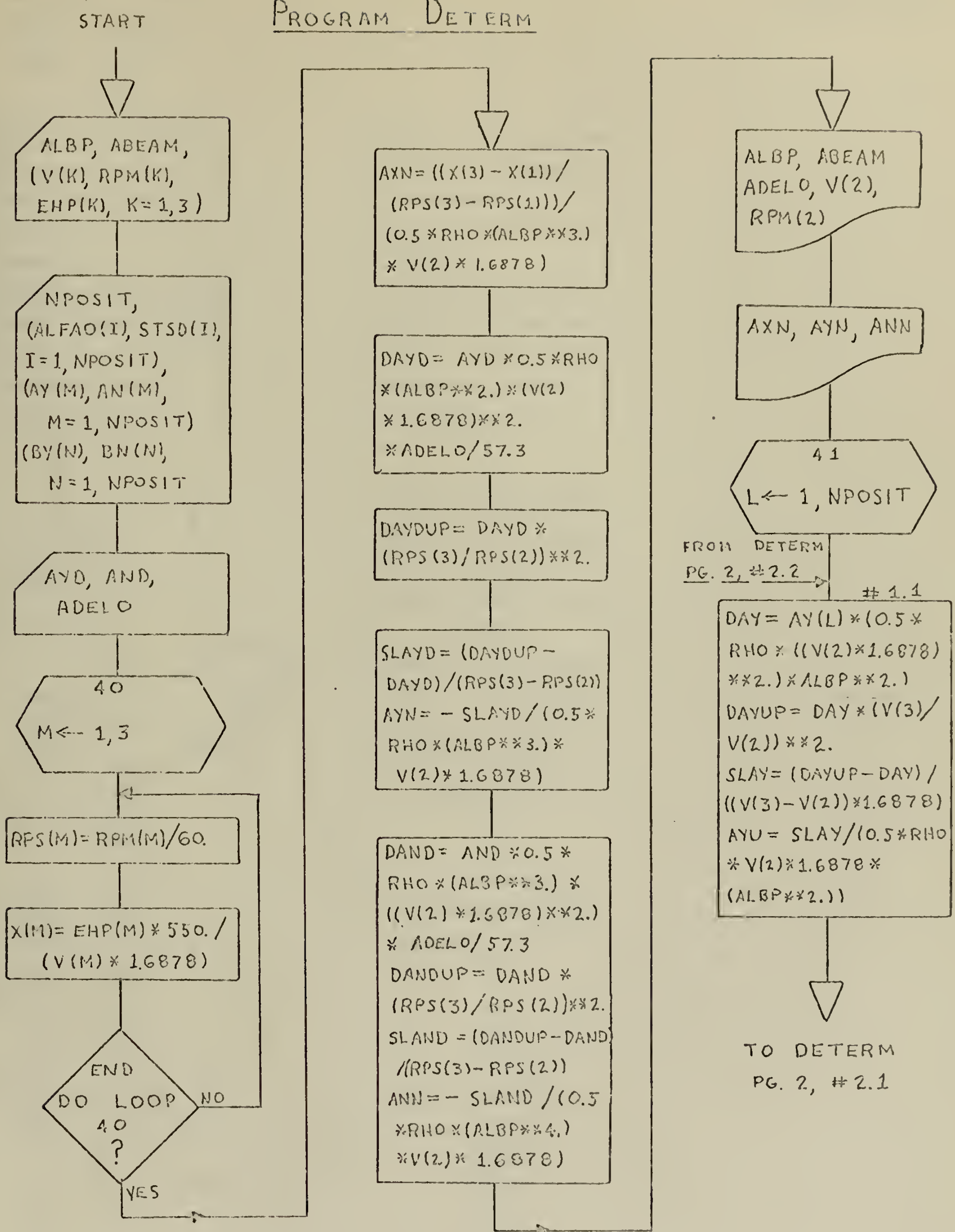
ITEM	PROGRAM SYMBOL	CARD	COLUMNS	FORMAT
LBP Ship A	ALBP	1	1-10	F10.2
Beam Ship A	ABEAM	1	11-20	F10.2
Speed at 1 kt. below equil. speed	V(1)	2	1-10	F10.2
RPM at this speed	RPM(1)	2	11-20	F10.2
EHP at this speed	EHP(1)	2	21-30	F10.2
Equil. speed (kts.)	V(2)	3	1-10	F10.2
RPM at equil.	RPM(2)	3	11-20	F10.2
EHP at equil.	EHP(2)	3	21-30	F10.2
Speed at 1 kt. above equil. speed	V(3)	4	1-10	F10.2
RPM at this speed	RPM(3)	4	11-20	F10.2
EHP at this speed	EHP(3)	4	21-30	F10.2
# positions examined	NPOSIT	5	1-4	I4
Long. separation at each position	ALFAO(I)	6	1-10	F10.2
Lat. separation at each position	STSD(I)	6	11-20	F10.2
(one card for each position)				
Y force at each position, Ship A	AY(M)	7	1-14	E14.4
N moment at each position, Ship A	AN(M)	7	15-28	E14.4
(one card for each position)				

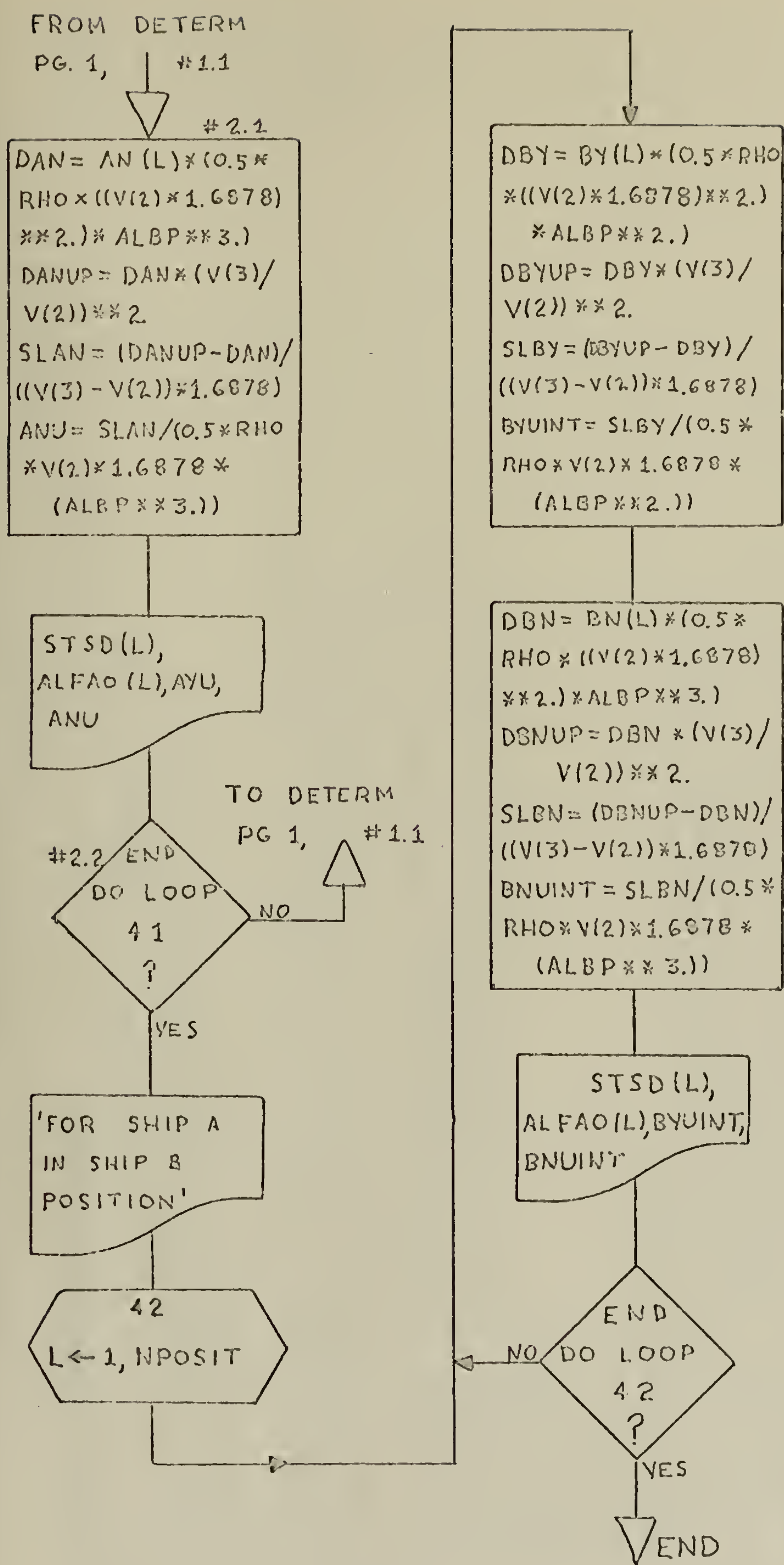
ITEM	PROGRAM SYMBOL	CARD	COLUMNS	FORMAT
Y force on Ship A in Ship B position	BY(N)	8	1-14	E14.4
N moment on Ship A in Ship B position	BN(N)	8	15-28	E14.4
	(one card for each position)			
Y_δ Ship A	AYDEL	9	1-14	E14.4
N_δ Ship A	ANDEL	9	15-28	E14.4
δ at equil, Ship A	ADELO	9	29-38	F10.2

The output format for DETERM is illustrated by the sample results section of this Appendix and should be self-explanatory. In the output, X_n is represented by DX/DN, Y_n by DY/DN, etc.

III. FLOW CHART

PROGRAM DETERM




FROM DETERM PG. 1, #1.1
#2.1
DAN= AN(L)*(0.5*RHO*((V(2)*1.6878)**2.)*ALBP**3.)
DANUP= DAN*(V(3)/V(2))**2.
SLAN= (DANUP-DAN)/((V(3)-V(2))*1.6878)
ANU= SLAN/(0.5*RHO*V(2)*1.6878*(ALBP**3.))
STSD(L), ALFAO(L),AYU, ANU
TO DETERM PG 1, #1.1
#2.2
END DO LOOP 41 ?
NO
YES
'FOR SHIP A IN SHIP B POSITION'
42
L←1, NPOSIT
DBY= BY(L)*(0.5*RHO*((V(2)*1.6878)**2.)*ALBP**2.)
DBYUP= DBY*(V(3)/V(2))**2.
SLBY=(DBYUP-DBY)/((V(3)-V(2))*1.6878)
BYUINT= SLBY/(0.5*RHO*V(2)*1.6878*(ALBP**2.))
DBN= BN(L)*(0.5*RHO*((V(2)*1.6878)**2.)*ALBP**3.)
DBNUP=DBN*(V(3)/V(2))**2.
SLBN= (DBNUP-DBN)/((V(3)-V(2))*1.6878)
BNUINT = SLBN/(0.5*RHO*V(2)*1.6878*(ALBP**3.))
STSD(L), ALFAO(L),BYUINT, BNUINT
END DO LOOP 42 ?
NO
YES
END

```
C  IV. DETERM  PROGRAM  LISTING
C
C  PROGRAM D E T E R M...TO DETERMINE THE VALUES OF SOME
C  HYDRODYNAMIC DERIVATIVES OTHER THAN THE INTERACTION
C  DERIVATIVES.  THOSE DETERMINED ARE DX/DN. DY/DN,
C  DN/DN,DY/DU AND DN/DU FOR SHIP A.
C
C  ALSO DETERMINED ARE DY/DU AND DN/DU FOR SHIP A WERE IT IN
C  SHIP B'S POSITION-----THESE WILL BE MODIFIED IN PROGRAM
C  SOLVE TO GET NUMERICAL VALUES FOR DY/DU AND DN/DU FOR B.
C
      DIMENSION V(3),RPM(3),RPS(3),EHP(3),X(3),AY(20),STSD(2
     10),ALFAO(20),AN(20),BY(20),BN(20)
      DATA FPS,FHP,FRPS,RHO,RAD/1.6878,550.,60.,1.9905,57.3/
  301 FORMAT(2F10.2)
  302 FORMAT(3F10.2)
  303 FORMAT(I4)
  304 FORMAT(2F14.4)
  305 FORMAT (2E12.4,F10.2)
  400 FORMAT(/////15X,' SHIP A ',/15X,'  LBP= ',F10.2,5X,
     1'  BEAM= ',F10.2,/15X,'  OPEN WATER RUDDER ANGLE= ',
     2F10.2,/15X,'  EQUIL SPEED IN KTS= ',F10.2,3X,
     3'  EQUIL RPM= ',F10.2)
  401 FORMAT(//,15X,' AT SIDE TO SIDE DIST= ',F10.2,/15X,
     1' ALFA ZERO= ',F10.2,3X,'  DY/DU= ',F12.4,/40X,
     2'  DN/DU= ', E12.4)
  402 FORMAT(//,15X,'DX/DN=',E14.4/15X,'DY/DN=',E14.4/15X,
     1'DN/DN=',E14.4)
  403 FORMAT('1',//////15X,'FOR SHIP A IN SHIP B POSITION')
C  READ SHIP A DIMENSIONS
      READ(5,301) ALBP,ABEAM
C  READ SHIP A EHP DATA NEAR EQUILIBRIUM SPEED.
      READ (5,302) (V(K),RPM(K),EHP(K),K=1,3)
C  NPOSIT=NUMBER OF POSITIONS FOR WHICH DY/DU AND DN/DU WILL
C  BE CALCULATED.
      READ(5,303) NPOSIT
```

```
C  READ Y FORCE AND N MOMENT ON SHIP A AND SIDE TO SIDE                 DTRM0037
C  SEPARATION,(STSD), AND LONGITUDINAL SEPARATION (ALFAD) AT            DTRM0038
C  NPOSIT POINTS, (UP TO 20).                                           DTRM0039
      READ(5,301)(ALFAD(I),STSD(I),I=1,NPOSIT)                          DTRM0040
      READ(5,304)(AY(M),AN(M),M=1,NPOSIT)                               DTRM0041
C  READ Y AND N VALUES FOR SHIP A IN SHIP B POSITION,                   DTRM0042
C  WITH SIGNS CHANGED.                                                  DTRM0043
      READ(5,304)(BY(N),BN(N),N=1,NPOSIT)                               DTRM0044
C  READ DY/D(DELTA) AND DN/D(DELTA) FOR SHIP A, (DELTA                  DTRM0045
C  IS RUDDER ANGLE IN RADIANS),....READ EQUILIBRIUM RUDDER              DTRM0046
C  ANGLE  (ADELO) IN DEGREES FOR SHIP A IN OPEN WATER.                  DTRM0047
C                                                                       DTRM0048
C                                                                       DTRM0049
      READ(5,305) AYDEL,ANDEL,ADELO                                     DTRM0050
C  DETERMINE DX/DN FOR SHIP A (AXN)                                     DTRM0051
      DO 40 M=1,3                                                       DTRM0052
      RPS(M)=RPM(M)/FRPS                                                DTRM0053
   40 X(M)=EHP(M)*FHP/(V(M)*FPS)                                        DTRM0054
      AXN=((X(3)-X(1))/(RPS(3)-RPS(1))) /(0.5*RHO*(ALBP**3.)           DTRM0055
     1*V(2)*FPS)                                                        DTRM0056
C  DETERMINE DY/DN FOR SHIP A (AYN)                                     DTRM0057
      DAYD=AYDEL*0.5*RHO*(ALBP**2.)*(V(2)*FPS)**2.*ADELO/RAD            DTRM0058
      DAYDUP=DAYD*(RPS(3)/RPS(2))**2.                                   DTRM0059
      SLAYD=(DAYDUP-DAYD)/(RPS(3)-RPS(2))                               DTRM0060
      AYN=-SLAYD/(0.5*RHO*(ALBP**3.)*V(2)*FPS)                          DTRM0061
C  DETERMINE DN/DN FOR SHIP A (ANN)                                     DTRM0062
      DAND=ANDEL*.5*RHO*(ALBP**3.)*(V(2)*FPS)**2.*ADELO/RAD             DTRM0063
      DANDUP=DAND*(RPS(3)/RPS(2))**2.                                   DTRM0064
      SLAND=(DANDUP-DAND)/(RPS(3)-RPS(2))                               DTRM0065
      ANN=-SLAND/(0.5*RHO*(ALBP**4.)*V(2)*FPS)                          DTRM0066
      WRITE(6,400) ALBP,AREAM,ADELO,V(2),RPM(2)                         DTRM0067
      WRITE(6,402)AXN,AYN,ANN                                           DTRM0068
C  DETERMINE DY/DU (AYU) AND DN/DU (ANU) FOR SHIP A AT                  DTRM0069
C  VARIOUS POSITIONS.                                                   DTRM0070
      DO 41 L=1,NPOSIT                                                  DTRM0071
      DAY=AY(L)*(0.5*RHO*((V(2)*FPS)**2.)*ALBP**2. )                    DTRM0072
```

```
      DAYUP=DAY*(V(3)/V(2))**2.                                         DTRM0073
      SLAY=(DAYUP-DAY)/((V(3)-V(2))*EPS)                                DTRM0074
      AYU=SLAY/(0.5*RHO*V(2)*EPS*(ALBP**2.))                            DTRM0075
      DAN=AN(L)*(0.5*RHO*((V(2)*EPS)**2.)*ALBP**3.)                     DTRM0076
      DANUP=DAN*(V(3)/V(2))**2.                                         DTRM0077
      SLAN=(DANUP-DAN)/((V(3)-V(2))*EPS)                                DTRM0078
      ANU=SLAN/(0.5*RHO*V(2)*EPS*(ALBP**2.))                            DTRM0079
      WRITE(6,401) STSD(L),ALFAO(L),AYU,ANU                             DTRM0080
   41 CONTINUE                                                          DTRM0081
      WRITE(6,403)                                                      DTRM0082
      DO 42 L=1,NPOSIT                                                  DTRM0083
      DBY=BY(L)*(0.5*RHO*((V(2)*EPS)**2.)*ALBP**2. )                    DTRM0084
      DBYUP=DBY*(V(3)/V(2))**2.                                         DTRM0085
      SLBY=(DBYUP-DBY)/((V(3)-V(2))*EPS)                                DTRM0086
      BYUINT=SLBY/(0.5*RHO*V(2)*EPS*(ALBP**2.))                         DTRM0087
      DBN=BN(L)*(0.5*RHO*((V(2)*EPS)**2.)*ALBP**3.)                     DTRM0088
      DBNUP=DBN*(V(3)/V(2))**2.                                         DTRM0089
      SLBN=(DBNUP-DBN)/((V(3)-V(2))*EPS)                                DTRM0090
      BNUINT=SLBN/(0.5*RHO*V(2)*EPS*(ALBP**2.))                         DTRM0091
      WRITE(6,401)STSD(L),ALFAO(L),BYUINT,BNUINT                        DTRM0092
   42 CONTINUE                                                          DTRM0093
      WRITE(6,900)                                                      DTRM0094
  900 FORMAT('1')                                                       DTRM0095
      STOP                                                              DTRM0096
      END                                                               DTRM0097
```

V. SAMPLE RESULTS

```
SHIP A
 LBP=      528.50        BEAM=       76.00
 OPEN WATER RUDDER ANGLE=        1.20
 EQUIL SPEED IN KTS=       15.00     EQUIL RPM=        65.00


DX/DN=     0.4623E-04
DY/DN=    -0.5236E-05
DN/DN=     0.2615E-05


 AT SIDE TO SIDE DIST=        60.00
 ALFA ZERO=          0.00    DY/DU=   0.1525E-02
                             DN/DU=  -0.6710E-03


 AT SIDE TO SIDE DIST=        90.00
 ALFA ZERO=          0.00    DY/DU=   0.1098E-02
                             DN/DU=  -0.4636E-03


 AT SIDE TO SIDE DIST=        70.00
 ALFA ZERO=        300.00    DY/DU=   0.3359E-03
                             DN/DU=   0.2255E-03


 AT SIDE TO SIDE DIST=        80.00
 ALFA ZERO=       -400.00    DY/DU=  -0.4762E-03
                             DN/DU=   0.1728E-03
```

FOR SHIP A IN SHIP B POSITION

```
AT SIDE TO SIDE DIST=      60.00
ALFA ZERO=          0.00   DY/DU=  -0.1525E-02
                           DN/DU=   0.6710E-03

AT SIDE TO SIDE DIST=      90.00
ALFA ZERO=          0.00   DY/DU=  -0.1098E-02
                           DN/DU=   0.4636E-03

AT SIDE TO SIDE DIST=      70.00
ALFA ZERO=        300.00   DY/DU=   0.3434E-03
                           DN/DU=   0.8686E-04

AT SIDE TO SIDE DIST=      80.00
ALFA ZERO=       -400.00   DY/DU=  -0.4290E-04
                           DN/DU=  -0.1671E-03
```

Appendix IV

PROGRAM SOLVE

I. Discussion

II. Input/Output Format

III. Flow Chart

IV. Program Listing

V. Sample Results

Appendix IV

PROGRAM SOLVE

I. Discussion

Program SOLVE, using the hydrodynamic derivative inputs for Ship A, and for Ship A in Ship B's position, calculates the full set of derivatives for Ship B. The derivatives for both ships are dimensionalized in the program, and the applicable coefficients for the variables of the equations of motion are computed. SOLVE then calls subroutine DETER which computes the values of the coefficients of the polynomial in σ which is the result of evaluating the determinant of the equations of motion coefficients.

Subroutine DETER, in the process of performing necessary polynomial addition, subtraction and multiplication, calls on library routines of the IBM Scientific Subroutine Package (SSP). These subroutines are PADD, PSUB and PMPY and are fully described in the IBM Application Program publication H20-0205-3, entitled "System/360 Scientific Subroutine Package, (360A-CM-03X) Version III". These routines are included in the program listings in this appendix and were used without modification.

Once DETER has evaluated the determinant to get the characteristic polynomial in σ, execution is returned to the main program, SOLVE, to determine the roots of this equation, which are the stability roots for the system. The roots are determined by a fourth SSP subroutine, POLRT, used without modification and listed in this appendix.

II. Input/Output Format

Program SOLVE's input is ordered as follows:

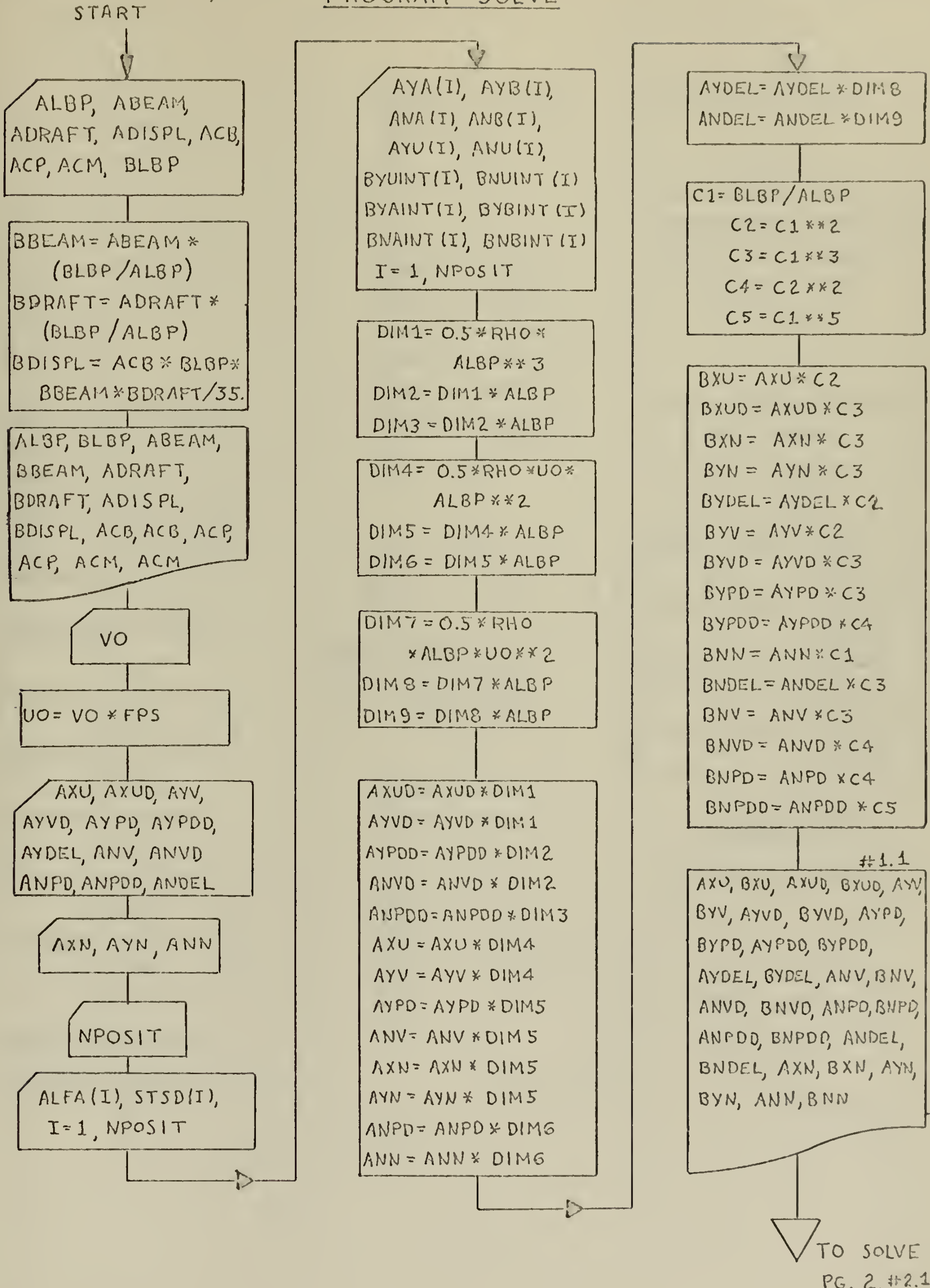
ITEM	PROGRAM SYMBOL	CARD	COLUMNS	FORMAT
Ship A Characteristics	ALBP, ABEAM, ADRAFT, ADISPL, ACB, ACP, ACM	1	1-70	7F10.2
Ship B Length	BLBP	2	1-10	F10.2
Equil. Spd. in kts.	VO	3	1-10	F10.2
Dimensionless open water derivatives, Ship A ($AXU = X_{u_A}$, etc.)	AXU, AXUD, AYU, AYVD, AYPD, AYPOD, AYDEL, ANV, ANVD, ANPD, ANPOD, ANDEL	4-15	1-14	E14.4
Dimensionless derivatives from DETERM	AXN, AYN, ANN	16-18	1-14	E14.4
# Positions examined	NPOSIT	19	1-4	I4
Long'l and Lat. separation at each position	ALFA(I), STSD(I)	20	1-20	2F10.2
	(one card for each position)			
Dimensionless Y_α, Y_β, N_α, N_β, Ship A	AYA(I), AYB(I), ANA(I), ANB(I)	21	1-56	4E14.4
	(one card for each position)			
Dimensionless Y_u, N_u, Ship A	AYU(I), ANU(I)	22	1-28	2E14.4
	(one card for each position)			
Dimensionless Y_u, N_u, Ship A, Ship B Position	BYUINT(I), BNUINT(I)	23	1-28	2E14.4
	(one card for each position)			

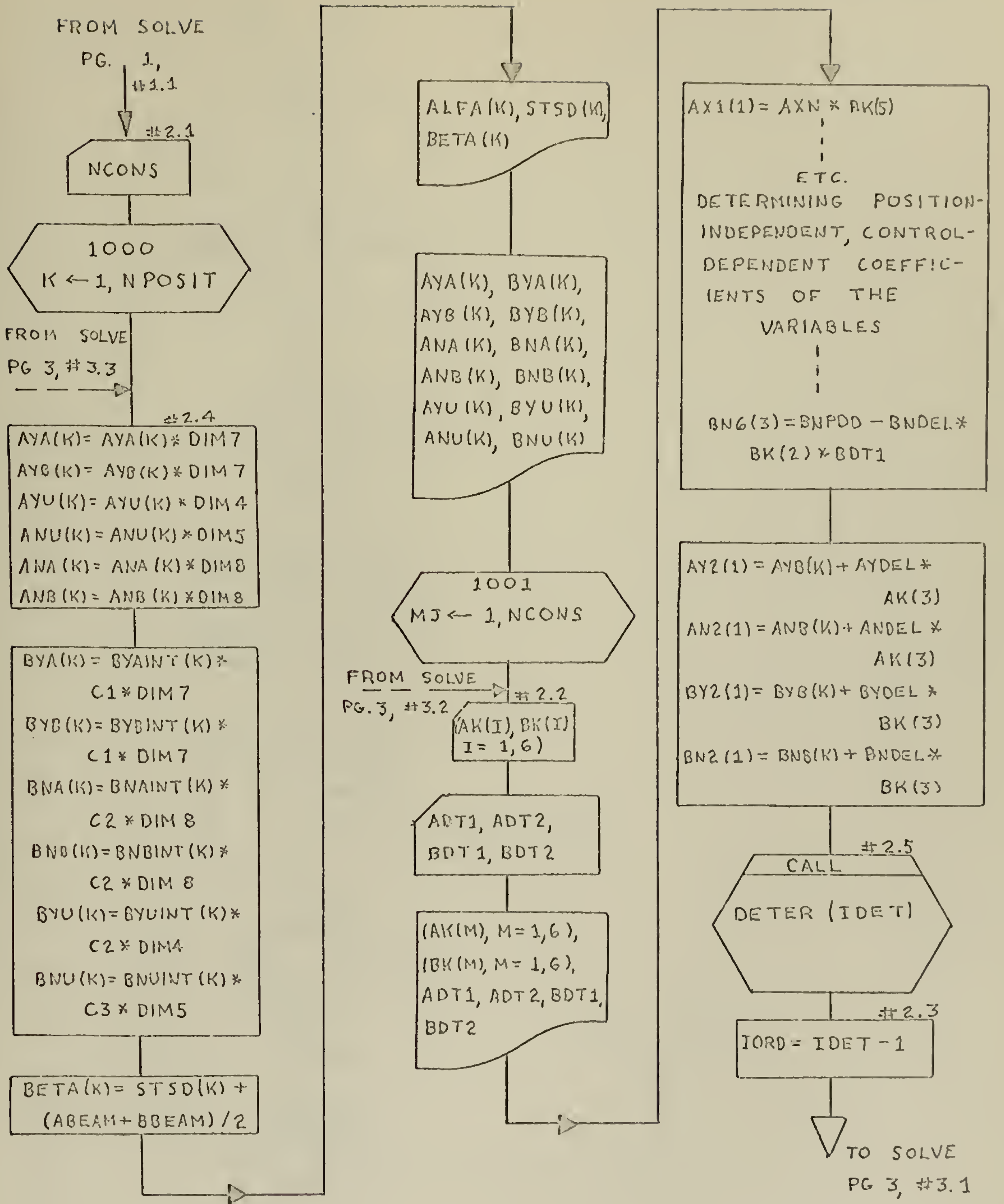
ITEM	PROGRAM SYMBOL	CARD	COLUMNS	FORMAT
Dimensionless Y_α, Y_β, Ship A, Ship B position	BYAINT(I), BYBINT(I)	24	1-28	2E14.4
	(one card for each position)			
Dimensionless N_α, N_β, Ship A, Ship B position	BNAINT(I), BNBINT(I)	25	1-28	2E14.4
	(one card for each position)			

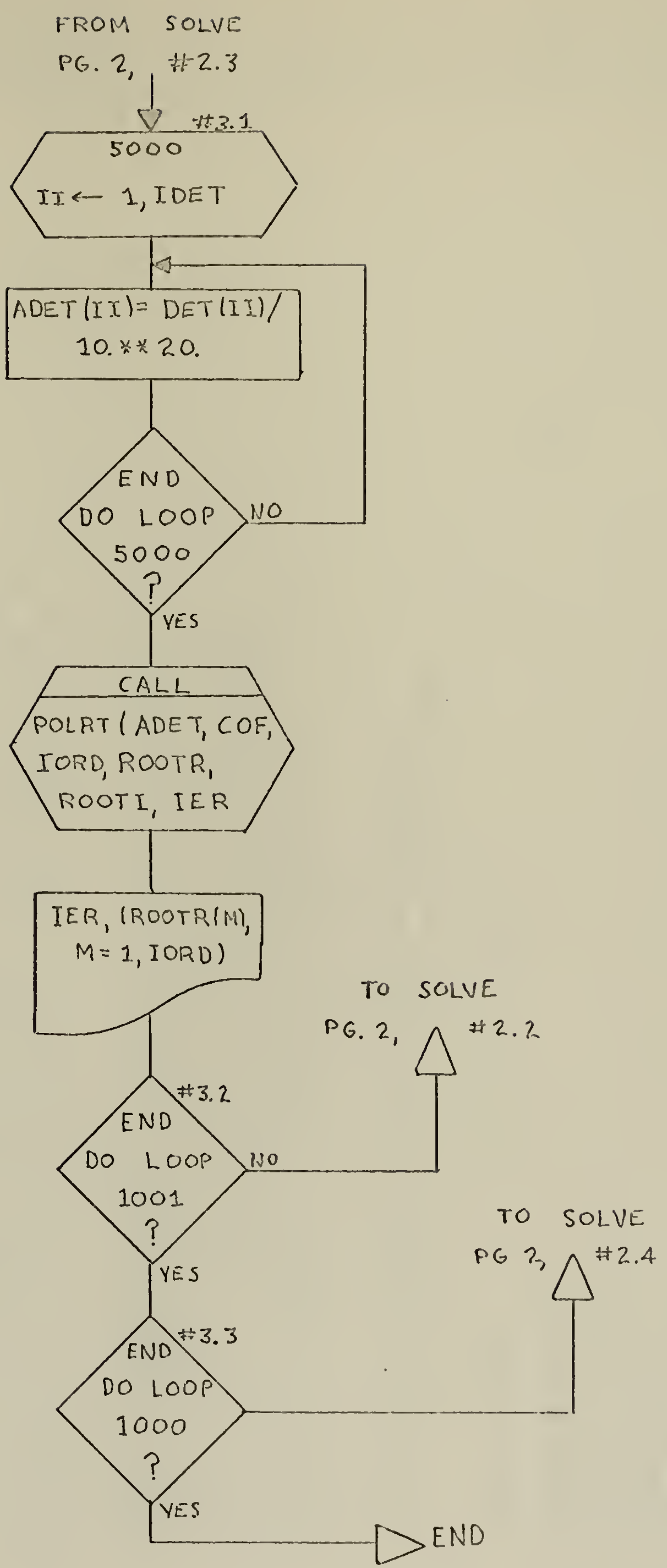
The output format for SOLVE is illustrated by the sample results section of this Appendix. In the output, Y_α is represented by DY/DA, etc.

III. SOLVE FLOW CHART

PROGRAM SOLVE




FROM SOLVE PG. 1, #1.1
#2.1
NCONS
1000
K ← 1, NPOSIT
FROM SOLVE PG 3, #3.3
#2.4
AYA(K)= AYA(K)* DIM7
AYB(K)= AYB(K)* DIM7
AYU(K)= AYU(K) * DIM4
ANU(K) = ANU(K)* DIM5
ANA(K)= ANA(K)* DIM8
ANB(K)= ANB(K)* DIM8
BYA(K)= BYAINT(K)* C1* DIM7
BYB(K)= BYBINT(K) * C1* DIM7
BNA(K)= BNAINT(K) * C2 * DIM 8
BNB(K)= BNBINT(K)* C2 * DIM 8
BYU(K)= BYUINT(K)* C2* DIM4
BNU(K)= BNUINT(K)* C3 * DIM5
BETA(K)= STSD(K) + (ABEAM+BBEAM)/2
ALFA(K), STSD(K), BETA(K)
AYA(K), BYA(K), AYB(K), BYB(K), ANA(K), BNA(K), ANB(K), BNB(K), AYU(K), BYU(K), ANU(K), BNU(K)
1001
MJ ← 1, NCONS
FROM SOLVE PG. 3, #3.2
#2.2
(AK(I), BK(I) I= 1, 6)
ADT1, ADT2, BDT1, BDT2
(AK(M), M= 1,6), (BK(M), M= 1,6), ADT1, ADT2, BDT1, BDT2
AX1(1)= AXN * AK(5)
ETC.
DETERMINING POSITION-INDEPENDENT, CONTROL-DEPENDENT COEFFICIENTS OF THE VARIABLES
BN6(3)=BNPOD − BNDEL* BK(2) * BDT1
AY2(1) = AYB(K)+ AYDEL* AK(3)
AN2(1) = ANB(K)+ ANDEL * AK(3)
BY2(1)= BYB(K)+ BYDEL * BK(3)
BN2(1)= BNB(K) + BNDEL* BK(3)
#2.5
CALL
DETER (IDET)
#2.3
IORD = IDET − 1
TO SOLVE PG 3, #3.1


FROM SOLVE
PG. 2, #2.3
#3.1
5000
II ← 1, IDET
ADET(II)= DET(II)/
10.**20.
END
DO LOOP
5000
?
NO
YES
CALL
POLRT (ADET, COF,
IORD, ROOTR,
ROOTI, IER
IER, (ROOTR(M),
M = 1, IORD)
TO SOLVE
PG. 2, #2.2
#3.2
END
DO LOOP
1001
?
NO
YES
TO SOLVE
PG 2, #2.4
#3.3
END
DO LOOP
1000
?
YES
END

```
C IV SOLVE   PROGRAM   LISTING
C
C  PROGRAM SOLVE...TAKES VALUES OF HYDRODYNAMIC DERIVATIVES
C  PREVIOUSLY DETERMINED FOR SHIP A AND SCALES THEM TO GET
C  VALUES FOR SHIP B.  THEN SUBROUTINE DETER IS CALLED TO
C  SOLVE THE DETERMINANT ARISING FROM THE SYSTEM OF EQNS
C  OF MOTION FOR THE TWO BODY SYSTEM.  THE SOLUTION OF THE
C  DETERMINANT WILL BE THE STABILITY CHARACTERISTIC EQN.
C
      DIMENSION AYA(10),AYB(10),ANA(10),ANB(10),AYU(10),
     1ANU(10),BYA(10),BYB(10),BNA(10),BNB(10),BYU(10),
     2BNU(10),AK(6),BK(6),ALFA(10),STSD(10),BETA(10),AX1(3),
     3AX3(3),AY2(3),AY4(3),AY5(3),AN2(3),AN4(3),AN5(3),
     4BX3(3),BY2(3),BY4(3),BY5(3),BN2(3),BN4(3),BN5(3),
     5BY1(3),BY6(3),BN6(3),DET(24),BYUINT(10),BNUINT(10),
     6BYAINT(10),BYBINT(10),BNAINT(10),BNBINT(10),
     7ROOTR(20),ROOTI(20),COF(20),ADET(24)
C
      COMMON AX1,AX3,AY2,AY4,AY5,AN2,AN4,AN5,DET,
     1       BX1,BX3,BY2,BY4,BY5,BN2,BN4,BN5,BY6,BN6
C
      DATA EPS,RHO        /1.6787,1.9905  /
C
  700 FORMAT(7F10.2)
  701 FORMAT(F10.2)
  702 FORMAT (F14.4)
  703 FORMAT(I4)
  704 FORMAT(4F14.4)
  705 FORMAT(2F14.4)
  706 FORMAT(2F10.2)
  707 FORMAT(6F10.2)
  708 FORMAT(4F10.2)
C
  743 FORMAT(/15X,'CHAR. EQN. REAL ROOTS ARE'/(24X,F14.4))
  744 FORMAT(  15X,'POLRT TEST VALUE IER= ',I4)
  745 FORMAT(  15X,'SHIP A TIME LAGS ARE',2(F10.2,2X),/
```

```
     115X,'SHIP B TIME LAGS ARE',2(F10.2,3X))                           SOLV0037
  746 FORMAT(  15X,'SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6'/            SOLV0038
     12(15X,3(F10.6,3X)/))                                              SOLV0039
  747 FORMAT('2',///,15X,'SHIP A CONTROL SYSTEM CONSTANTS'              SOLV0040
     1' 1 TO 6'/2(15X,3(F10.6,3X)/))                                    SOLV0041
  748 FORMAT(/,29X,'SHIP A',15X,'SHIP B',/                              SOLV0042
     115X,'DY/DA',5X,E14.4,7X,E14.4/                                    SOLV0043
     215X,'DY/DB',5X,E14.4,7X,E14.4/                                    SOLV0044
     315X,'DN/DA',5X,E14.4,7X,E14.4/                                    SOLV0045
     415X,'DN/DB',5X,E14.4,7X,E14.4/                                    SOLV0046
     515X,'DY/DU',5X,E14.4,7X,E14.4/                                    SOLV0047
     615X,'DN/DU',5X,E14.4,7X,E14.4/ )                                  SOLV0048
C                                                                       SOLV0049
  749 FORMAT(       / /15X,'AT ALFA= ',F7.1,' STSP= ',F7.1,             SOLV0050
     1' AND BETA= ',F7.1/15X,' THE POSITION DEPENDENT DER'              SOLV0051
     2'IVATIVES ARE')                                                   SOLV0052
C                                                                       SOLV0053
  750 FORMAT(/////,26X,'SHIP A',15X,'SHIP B'/                           SOLV0054
     115X,'LBP=',        7X,F10.2,11X,F10.2/                            SOLV0055
     215X,'BEAM=',       6X,F10.2,11X,F10.2/                            SOLV0056
     315X,'DRAFT=',      5X,F10.2,11X,F10.2/                            SOLV0057
     415X,'DISPL=',      5X,F10.2,11X,F10.2/                            SOLV0058
     515X,'CB=',         8X,F10.4,11X,F10.4/                            SOLV0059
     615X,'CP=',         8X,F10.4,11X,F10.4/                            SOLV0060
     715X,'CM=',         8X,F10.4,11X,F10.4)                            SOLV0061
C                                                                       SOLV0062
  751 FORMAT(////,26X,'SHIP A',15X,'SHIP B'/12X,'DX/DU',5X,             SOLV0063
     1E14.4,7X,E14.4/12X,'DX/DUDOT',2X,E14.4,7X,E14.4/12X,              SOLV0064
     2'DY/DV',5X,E14.4,7X,E14.4/12X,'DY/DVDOT',2X,E14.4,7X,E            SOLV0065
     314.4/12X,'DY/DPDOT',3X,E14.4,7X,E14.4/12X,'DY/DPDDOT',            SOLV0066
     41X,E14.4,7X,E14.4,/12X,'DY/DDEL',3X,E14.4,7X,E14.4,/              SOLV0067
     512X,'DN/DV',5X,E14.4,7X,E14.4/12X,'DN/DVDOT',2X,E14.4,            SOLV0068
     67X,E14.4/12X,'DN/DPDOT',2X,E14.4,7X,E14.4,/12X,                   SOLV0069
     7'DN/DPDDOT',1X,E14.4,7X,E14.4,/12X,'DN/DDEL',3X,E14.4,            SOLV0070
     87X,E14.4,/12X,'DX/DN',5X,E14.4,7X,E14.4/12X,'DY/DN',5X            SOLV0071
     9,E14.4,7X,E14.4/12X,'DN/DN',5X,E14.4,7X,E14.4)                    SOLV0072
```

```
C
  752 FORMAT(    //,15X,' THE DETERMINANT IN ASCENDING D'/
     115X,' POWERS IS',/(24X,E14.4))
C
      READ(5,700)ALBP,ABEAM,ADRAFT,ADISPL,ACB,ACP,ACM
      READ(5,701)BLBP
C  COMPUTE SHIP B CHARACTERISTICS
      BBEAM=ABEAM*BLBP/ALBP
      BDRAFT=ADRAFT*BBEAM/ABEAM
      BDISPL=ACB*BLBP*BBEAM*BDRAFT/35.
      WRITE(6,750)ALBP,BLBP,ABEAM,BBEAM,ADRAFT,BDRAFT,ADISPL
     1,BDISPL,ACB,ACB,ACP,ACP,ACM,ACM
C  VO IS EQUILIBRIUM SPEED IN KTS, UO IN FT/SEC
      READ(5,701) VO
      UO=VO*FPS
C
C  READ SHIP A POSITION-INDEPENDENT DERIVATIVES, USING
C  STROM-TEJSEN VALUES IN NON-DIMENSIONAL FORM,
      READ(5,702)AXU,AXUD,AYV,AYVD,AYPD,AYPDD,AYDEL,ANV,ANVD
     1,ANPD,ANPDD,ANDEL
C  THESE DERIVATIVES FROM PROGRAM DETERM ARE ALSO POSITION-
C  INDEPENDENT AND DIMENSIONLESS
      READ(5,702) AXN,AYN,ANN
C
C  READ POSITION-DEPENDENT DERIVATIVES IN NON-DIMENSIONAL
C  FORM,  AYA,AYB,ANA,ANB ARE FROM PROGRAM INTCOF.
C  AYU,ANU,BYUINT AND BNUINT FROM PROGRAM DETERM
C  BYAINT,BYBINT,BNAINT AND BNBINT FROM PROGRAM INTCOF. BY
C  ASSUMING SHIP A IN SHIP B POSITION.
C  NPOSIT IS THE NO. OF POSITIONS OF INTEREST (UP TO 10)
C
      READ(5,703)NPOSIT
      READ(5,706)(ALFA(I),STSD(I),I=1,NPOSIT)
      READ(5,704)(AYA(I),AYB(I),ANA(I),ANB(I),I=1,NPOSIT)
      READ(5,705)(AYU(I),ANU(I),I=1,NPOSIT)
      READ(5,705)(BYUINT(I),BNUINT(I),I=1,NPOSIT)
```

```
      READ(5,705)(BYAINT(K),BYRINT(K),K=1,NPOSIT)                    SOLV0109
      READ(5,705)(BNAINT(K),BNRINT(K),K=1,NPOSIT)                    SOLV0110
C                                                                    SOLV0111
C     CALCULATE DIMENSIONALIZING FACTORS FOR HYDRO DERIVATIVES       SOLV0112
C                                                                    SOLV0113
      DIM1=0.5*RHO*ALBP**3.                                          SOLV0114
      DIM2=DIM1*ALBP                                                 SOLV0115
      DIM3=DIM2*ALBP                                                 SOLV0116
      DIM4=0.5*RHO*UO*ALBP**2.                                       SOLV0117
      DIM5=DIM4*ALBP                                                 SOLV0118
      DIM6=DIM5*ALBP                                                 SOLV0119
      DIM7=0.5*RHO*ALBP*UO**2.                                       SOLV0120
      DIM8=DIM7*ALBP                                                 SOLV0121
      DIM9=DIM8*ALBP                                                 SOLV0122
C                                                                    SOLV0123
C     GET SHIP A POSITION INDEPENDENT HYDRO DERIVATIVES IN           SOLV0124
C     DIMENSIONAL FORM                                               SOLV0125
C                                                                    SOLV0126
      AXUD=AXUD*DIM1                                                 SOLV0127
      AYVD=AYVD*DIM1                                                 SOLV0128
      AYPDD=AYPDD*DIM2                                               SOLV0129
      ANVD=ANVD*DIM2                                                 SOLV0130
      ANPDD=ANPDD*DIM3                                               SOLV0131
      AXU=AXU*DIM4                                                   SOLV0132
      AYV=AYV*DIM4                                                   SOLV0133
      AYPD=AYPD*DIM5                                                 SOLV0134
      ANV=ANV*DIM5                                                   SOLV0135
      AXN=AXN*DIM5                                                   SOLV0136
      AYN=AYN*DIM5                                                   SOLV0137
      ANPD=ANPD*DIM6                                                 SOLV0138
      ANN=ANN*DIM6                                                   SOLV0139
      AYDEL=AYDEL*DIM8                                               SOLV0140
      ANDEL=ANDEL*DIM9                                               SOLV0141
C                                                                    SOLV0142
C     CALCULATE CONVERSION FACTORS TO SCALE SHIP A DIMENSIONAL       SOLV0143
C     HYDRODYNAMIC DERIVATIVE VALUES TO THOSE FOR SHIP B             SOLV0144
```

```
C                                                                       SOLV0145
      C1=BLBP/ALBP                                                      SOLV0146
      C2=C1**2.                                                         SOLV0147
      C3=C1**3.                                                         SOLV0148
      C4=C2**2.                                                         SOLV0149
      C5=C1**5.                                                         SOLV0150
C                                                                       SOLV0151
C  DETERMINING SHIP B'S POSITION-INDEPENDENT DERIVATIVES.               SOLV0152
C  IN DIMENSIONAL FORM                                                  SOLV0153
C                                                                       SOLV0154
      BXU  =AXU  *C2                                                    SOLV0155
      BXUD =AXUD *C3                                                    SOLV0156
      BXN  =AXN  *C3                                                    SOLV0157
      BYN  =AYN  *C3                                                    SOLV0158
      BYDEL=AYDEL*C2                                                    SOLV0159
      BYV  =AYV  *C2                                                    SOLV0160
      BYVD =AYVD *C3                                                    SOLV0161
      BYPD =AYPD *C3                                                    SOLV0162
      BYPDD=AYPDD*C4                                                    SOLV0163
      BNN  =ANN  *C1                                                    SOLV0164
      BNDEL=ANDEL*C3                                                    SOLV0165
      BNV  =ANV  *C3                                                    SOLV0166
      BNVD =ANVD *C4                                                    SOLV0167
      BNPD =ANPD *C4                                                    SOLV0168
      BNPDD=ANPDD*C5                                                    SOLV0169
      WRITE(6,751)AXU,BXU,AXUD,BXUD,AYV,BYV,AYVD,BYVD,AYPD,             SOLV0170
     1BYPD,AYPDD,BYPDD,AYDEL,BYDEL,ANV,BNV,ANVD,BNVD,ANPD,              SOLV0171
     2BNPD,ANPDD,BNPDD,ANDEL,BNDEL,AXN,BXN,AYN,BYN,ANN,BNN              SOLV0172
C                                                                       SOLV0173
C  NCONS IS THE NUMBER OF SETS OF CONTROL SYSTEM CONSTANTS              SOLV0174
C  TO BE USED.  ANY NUMBER OF SETS OF CONSTANTS, SIX PER                SOLV0175
C  SHIP, MAY BE USED                                                    SOLV0176
C                                                                       SOLV0177
      READ(5,703) NCONS                                                 SOLV0178
C                                                                       SOLV0179
C  DO LOOP CYCLES CALCULATION OF CONTROL-DEPENDENT PAR-                 SOLV0180
```

```
C   AMETERS ONCE FOR EACH SET OF CONTROL SYSTEM CONSTANTS              SOLV0181
C   BEING EXAMINED AT EACH POSITION.                                   SOLV0182
C                                                                      SOLV0183
      DO 1000 K=1.NPOSIT                                               SOLV0184
C                                                                      SOLV0185
C   DIMENSIONALIZE THE POSITION-DEPENDENT DERIVATIVES FOR              SOLV0186
C   SHIP A                                                             SOLV0187
      AYA(K)=AYA(K)*DIM7                                               SOLV0188
      AYB(K)=AYB(K)*DIM7                                               SOLV0189
      AYU(K)=AYU(K)*DIM4                                               SOLV0190
      ANU(K)=ANU(K)*DIM5                                               SOLV0191
      ANA(K)=ANA(K)*DIM8                                               SOLV0192
      ANB(K)=ANB(K)*DIM8                                               SOLV0193
C   EVALUATE POSITION-DEPENDENT DERIVATIVES FOR SHIP B                 SOLV0194
      BYA(K)=BYAINT(K)*C1*DIM7                                         SOLV0195
      BYB(K)=-BYBINT(K)*C1*DIM7                                        SOLV0196
      BNA(K)=BNAINT(K)*C2 *DIM8                                        SOLV0197
      BNB(K)=-BNBINT(K)*C2*DIM8                                        SOLV0198
      BYU(K)=BYUINT(K)*C2*DIM4                                         SOLV0199
      BNU(K)= BNUINT(K)*C3*DIM5                                        SOLV0200
C                                                                      SOLV0201
C   WRITE THE POSITION DEPENDENT DERIVATIVES FOR BOTH SHIPS.           SOLV0202
      BETA(K)=STSD(K)+(ABEAM+BBEAM)/2.                                 SOLV0203
      WRITE(6,749)ALFA(K),STSD(K),BETA(K)                              SOLV0204
      WRITE(6,748)AYA(K).BYA(K),AYB(K),BYB(K),ANA(K).BNA(K),           SOLV0205
     1ANB(K),BNB(K),AYU(K),BYU(K),ANU(K),BNU(K)                        SOLV0206
      DO 1001 MJ=1,NCONS                                               SOLV0207
C                                                                      SOLV0208
C   READ CONTROL SYSTEM CONSTANTS AND TIME LAGS                        SOLV0209
C                                                                      SOLV0210
      READ(5,707)(AK(I),I=1,6)                                         SOLV0211
      READ(5,707)(BK(I).I=1,6)                                         SOLV0212
      READ(5,708)ADT1.ADT2,BDT1.BDT2                                   SOLV0213
C   WRITE CONTROL SYSTEM CONSTANTS AND TIME LAGS                       SOLV0214
      WRITE(6,747)(AK(M),M=1,6)                                        SOLV0215
      WRITE(6,746)(BK(M),M=1,6)                                        SOLV0216
```

```
      WRITE(6,745)ADT1,ADT2,BDT1,BDT2
C
C  DETERMINING POSITION-INDEPENDENT BUT CONTROL-DEPENDENT
C  COEFFICIENTS OF THE VARIABLES.
      AX1(1)=AXN*AK(5)
      AX1(2)=AXN*(AK(6)-AK(5)*ADT2)
      AX1(3)=-AXN*AK(6)*ADT2
      AX3(1)=AXU
      AX3(2)=AXUD
      AY2(2)=AYDEL*(AK(4)-AK(3)*ADT1)
      AY2(3)=-AYDEL*AK(4)*ADT1
      AY4(1)=AYV
      AY4(2)=AYVD
      AY5(1)=AYDEL*AK(1)
      AY5(2)=AYPD+AYDEL*(AK(2)-AK(1)*ADT1)
      AY5(3)=AYPDD-AYDEL*AK(2)*ADT1
      AN2(2)=ANDEL*(AK(4)-AK(3)*ADT1)
      AN2(3)=-ANDEL*AK(4)*ADT1
      AN4(1)=ANV
      AN4(2)=ANVD
      AN5(1)=ANDEL*AK(1)
      AN5(2)=ANPD+ANDEL*(AK(2)-AK(1)*ADT1)
      AN5(3)=ANPDD-ANDEL*AK(2)*ADT1
C
C  SHIP B
      BX1(1)=BXN*BK(5)
      BX1(2)=BXN*(BK(6)-BK(5)*BDT2)+BXU
      BX1(3)= BXUD-BXN*BK(6)*BDT2
      BX3(1)=BXU
      BX3(2)=BXUD
      BY2(2)=BYDEL*(BK(4)-BK(3)*BDT1)+BYV
      BY2(3)=BYVD-BYDEL*BK(4)*BDT1
      BY4(1)=BYV
      BY4(2)=BYVD
      BY5(1)=BYV*U0
      BY5(2)=BYVD*U0
```

```
      BY6(1)=BYDEL*BK(1)-BYV*U0
      BY6(2)=BYPD+BYDEL*(BK(2)-BK(1)*BDT1)-BYVD*U0
      BY6(3)=BYPDD-BYDEL*BK(2)*BDT1
      BN2(2)=BNDEL*(BK(4)-BK(3)*BDT1)+BNV
      BN2(3)=BNVD-BNDEL*BK(4)*BDT1
      BN4(1)=BNV
      BN4(2)=BNVD
      BN5(1)=BNV*U0
      BN5(2)=BNVD*U0
      BN6(1)=BNDEL*BK(1)-BNV*U0
      BN6(2)=BNPD+BNDEL*(BK(2)-BK(1)*BDT1)-BNVD*U0
      BN6(3)=BNPDD-BNDEL*BK(2)*BDT1
C
C  DETERMINE POSITION- AND CONTROL-DEPENDENT COEFFICIENTS
C  OF THE VARIABLES
C
      AY2(1)=AYB(K)+AYDEL*AK(3)
      AN2(1)=ANB(K)+ANDEL*AK(3)
      BY2(1)=BYB(K)+BYDEL*BK(3)
      BN2(1)=BNB(K)+BNDEL*BK(3)
C
      CALL DETER (IDET)
      IORD=IDET-1
      DO 5000 II=1,IDET
      ADET(II)=DET(II)/10.**20.
 5000 CONTINUE
      CALL POLRT(ADET,COF,IORD,ROOTR,ROOTI,IER)
C
C  POLRT IS AN IBM SCIENTIFIC SUBROUTINE PACKAGE, (SSP).
C  ROUTINE WHICH, WHEN CALLED WITH THE
C  VECTOR OF COEFFICIENTS OF A POLYNOMIAL ,ARRANGED FROM
C  LOWEST TO HIGHEST ORDER, RETURNS THE REAL (ROOTR) AND
C  IMAGINARY (ROOTI) ROOTS OF THE POLYNOMIAL. COF IS A DUMMY
C  VECTOR,  IER IS A TEST VALUE WHICH EQUALS ZERO WHEN NO
C  DISCREPANCIES EXIST,
C
```

```
      WRITE(6,744)IER                                                    SOLV0289
      WRITE(6,743)(ROOTR(M),M=1,IORD)                                    SOLV0290
 1001 CONTINUE                                                           SOLV0291
 1000 CONTINUE                                                           SOLV0292
      STOP                                                               SOLV0293
      END                                                                SOLV0294
```

```
      SUBROUTINE DETER (IDET)
C
      DIMENSION AX1(3),AX3(3),AY2(3),AY4(3),AY5(3),AN2(3),
     1AN4(3),AN5(3),BX1(3),BX3(3),BY2(3),BY5(3),BN2(3),
     2BN4(3),BN5(3),BY6(3),BN6(3),BY4(3),
     3AX1BY3(6),AX3BX1(6),FAC(12),FFAC3(24),
     4BN6BY5(6),BY6BN5(6),FAC1(12),SUM1(24),
     5BN6BY2(6),BY6BN2(6),FAC3(12),SUM(24),
     6BN6BY4(6),BY6BN4(6),FAC5(12),DET(24),
     7AY4AN2(6),AN4AY2(6),FAC2(12),
     8AY5AN4(6),AN5AY4(6),FAC4(12),
     9AY2AN5(6),AN2AY5(6),FAC6(12),FFAC1(24),FFAC2(24)
      COMMON AX1,AX3,AY2,AY4,AY5,AN2,AN4,AN5,DET,
     1       BX1,BX3,BY2,BY4,BY5,BN2,BN4,BN5,BY6,BN6
C
C  EVALUATING THE DETERMINANT
C
C  DETERMINANT (DET)=(AX1*BX3-BX1*AX3)
C  *((BN6*BY5-BY6*BN5)*(AY4*AN2-AN4*AY2)
C  +(BN6*BY2-BY6*BN2)*(AY5*AN4-AN5*AY4)
C  +(BN6*BY4-BY6*BN4)*(AY2*AN5-AN2*AY5))
C
C  THIS IS SHORTENED TO SYMBOLS AS
C  DET=FAC*(FAC1*FAC2+FAC3*FAC4+FAC5*FAC6)
C
C  EVALUATING FAC
C
      CALL PMPY(AX1BX3,IRES1,AX1,3,BX3,2)
C
C  PMPY IS AN IBM SCIENTIFIC SUBROUTINE PACKAGE (SSP) SUB-
C  ROUTINE WHICH IS CALLED BY CALL PMPY (Z,IDIMZ,X,IDIMX,Y,
C  IDIMY) AND MULTIPLIES TWO POLYNOMIALS...Z=X*Y.  IDIMX,
C  IDIMY AND IDIMZ ARE THE DIMENSIONS OF THE RESPECTIVE
C  POLYNOMIALS, WHERE DIMENSION = DEGREE+1.
C
      CALL PMPY(AX3BX1,IRES2,AX3,2,BX1,3)
```

```
C                                                                       DETR0037
C  PSUB IS AN SSP SUBROUTINE THAT SUBRACTS ONE POLYNOMIAL               DETR0038
C  FROM ANOTHER.   CALL PSUB(Z,IDIMZ,X,IDIMX,Y,IDIMY)                   DETR0039
C  Z=X-Y                                                                DETR0040
      CALL PSUB(FAC,IFAC,AX1BX3,IRES1,AX3BX1,IRES2)                     DETR0041
C  EVALUATING FAC1                                                      DETR0042
      CALL PMPY(BN6BY5,IRES3,BN6,3,BY5,2)                               DETR0043
      CALL PMPY(BY6BN5,IRES4,BY6,3,BN5,2)                               DETR0044
      CALL PSUB(FAC1,IFAC1,BN6BY5,IRES3,BY6BN5,IRES4)                   DETR0045
      MFAC=1                                                            DETR0046
C  EVALUATING FAC2                                                      DETR0047
      CALL PMPY(AY4AN2,IRES5,AY4,2,AN2,3)                               DETR0048
      CALL PMPY(AN4AY2,IRES6,AN4,2,AY2,3)                               DETR0049
      CALL PSUB(FAC2,IFAC2,AY4AN2,IRES5,AN4AY2,IRES6)                   DETR0050
      MFAC=MFAC+1                                                       DETR0051
C  EVALUATING FAC3                                                      DETR0052
      CALL PMPY(BN6BY2,IRES7,BN6,3,BY2,3)                               DETR0053
      CALL PMPY(BY6BN2,IRES8,BY6,3,BN2,3)                               DETR0054
      CALL PSUB(FAC3,IFAC3,BN6BY2,IRES7,BY6BN2,IRES8)                   DETR0055
      MFAC=MFAC+1                                                       DETR0056
C  EVALUATING FAC4                                                      DETR0057
      CALL PMPY(AY5AN4,IRES9,AY5,3,AN4,2)                               DETR0058
      CALL PMPY(AN5AY4,IRES10,AN5,3,AY4,2)                              DETR0059
      CALL PSUB(FAC4,IFAC4,AY5AN4,IRES9,AN5AY4,IRES10)                  DETR0060
      MFAC=MFAC+1                                                       DETR0061
C  EVALUATING FAC5                                                      DETR0062
      CALL PMPY(BN6BY4,IRES11,BN6,3,BY4,2)                              DETR0063
      CALL PMPY(BY6BN4,IRES12,BY6,3,BN4,2)                              DETR0064
      CALL PSUB(FAC5,IFAC5,BN6BY4,IRES11,BY6BN4,IRES12)                 DETR0065
      MFAC=MFAC+1                                                       DETR0066
C  EVALUATING FAC6                                                      DETR0067
      CALL PMPY(AY2AN5,IRES13,AY2,3,AN5,3)                              DETR0068
      CALL PMPY(AN2AY5,IRES14,AN2,3,AY5,3)                              DETR0069
      CALL PSUB(FAC6,IFAC6,AY2AN5,IRES13,AN2AY5,IRES14)                 DETR0070
      MFAC=MFAC+1                                                       DETR0071
C                                                                       DETR0072
```

```
C  EVALUATING FAC1*FAC2, FAC3*FAC4, AND FAC5*FAC6
      CALL PMPY(FFAC1,IRES15,FAC1,IFAC1,FAC2,IFAC2)
      CALL PMPY(FFAC2,IRES16,FAC3,IFAC3,FAC4,IFAC4)
      CALL PMPY(FFAC3,IRES17,FAC5,IFAC5,FAC6,IFAC6)
C
C  EVALUATING THE SUMS
C  PADD IS AN SSP SUBROUTINE THAT ADDS 2 POLYNOMIALS
C  CALL PADD(Z,IDIMZ,X,IDIMX,Y,IDIMY)
C  Z=X+Y
C
      CALL PADD(SUM1,ISUM1,FFAC1,IRES15,FFAC2,IRES16)
      CALL PADD(SUM,ISUM,SUM1,ISUM1,FFAC3,IRES17)
C  MULTIPLY FAC*SUM, TO GET DETERMINANT, DET
      CALL PMPY(DET,IDET,FAC,IFAC,SUM,ISUM)
      RETURN
      END
```

```
      SUBROUTINE PSUB(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PSUB 400 PSUB0001
C   THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE                          PSUB0002
C        BEFORE COMPILING THIS UNDER  IBM FORTRAN G.                            PSUB0003
C                                                                       PSUB  10 PSUB0004
C     ..................................................................PSUB  20 PSUB0005
C                                                                       PSUB  30 PSUB0006
C        SUBROUTINE PSUB                                                PSUB  40 PSUB0007
C                                                                       PSUB  50 PSUB0008
C        PURPOSE                                                        PSUB  60 PSUB0009
C           SUBTRACT ONE POLYNOMIAL FROM ANOTHER                        PSUB  70 PSUB0010
C                                                                       PSUB  80 PSUB0011
C        USAGE                                                          PSUB  90 PSUB0012
C           CALL PSUB(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PSUB 100 PSUB0013
C                                                                       PSUB 110 PSUB0014
C        DESCRIPTION OF PARAMETERS                                      PSUB 120 PSUB0015
C           Z     - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM      PSUB 130 PSUB0016
C                   SMALLEST TO LARGEST POWER                           PSUB 140 PSUB0017
C           IDIMZ - DIMENSION OF Z (CALCULATED)                         PSUB 150 PSUB0018
C           X     - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDEREDPSUB 160 PSUB0019
C                   FROM SMALLEST TO LARGEST POWER                      PSUB 170 PSUB0020
C           IDIMX - DIMENSION OF X (DEGREE IS IDIMX-1)                  PSUB 180 PSUB0021
C           Y     - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,       PSUB 190 PSUB0022
C                   ORDERED FROM SMALLEST TO LARGEST POWER              PSUB 200 PSUB0023
C           IDIMY - DIMENSION OF Y (DEGREE IS IDIMY-1)                  PSUB 210 PSUB0024
C                                                                       PSUB 220 PSUB0025
C        REMARKS                                                        PSUB 230 PSUB0026
C           VECTOR Z MAY BE IN SAME LOCATION AS EITHER VECTOR X OR      PSUB 240 PSUB0027
C           VECTOR Y ONLY IF THE DIMENSION OF THAT VECTOR IS NOT LESS   PSUB 250 PSUB0028
C           THAN THE OTHER INPUT VECTOR                                 PSUB 260 PSUB0029
C           THE RESULTANT POLYNOMIAL MAY HAVE TRAILING ZERO COEFFICIENTSPSUB 270 PSUB0030
C                                                                       PSUB 280 PSUB0031
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  PSUB 290 PSUB0032
C           NONE                                                        PSUB 300 PSUB0033
C                                                                       PSUB 310 PSUB0034
C        METHOD                                                         PSUB 320 PSUB0035
C           DIMENSION OF RESULTANT VECTOR IDIMZ IS CALCULATED AS THE    PSUB 330 PSUB0036
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C           LARGER OF THE TWO INPUT VECTOR DIMENSIONS. COEFFICIENTS IN  PSUB 340 PSUB0037
C           VECTOR Y ARE THEN SUBTRACTED FROM CORRESPONDING COEFFICIENTSPSUB 350 PSUB0038
C           IN VECTOR X.                                                PSUB 360 PSUB0039
C                                                                       PSUB 370 PSUB0040
C     ..................................................................PSUB 380 PSUB0041
C                                                                       PSUB 390 PSUB0042
      DIMENSION Z(1),X(1),Y(1)                                          PSUB 410 PSUB0043
C                                                                       PSUB 420 PSUB0044
C     TEST DIMENSIONS OF SUMMANDS                                       PSUB 430 PSUB0045
C                                                                       PSUB 440 PSUB0046
      NDIM=IDIMX                                                        PSUB 450 PSUB0047
      IF (IDIMX-IDIMY) 10,20,20                                         PSUB 460 PSUB0048
   10 NDIM=IDIMY                                                        PSUB 470 PSUB0049
   20 IF (NDIM) 90,90,30                                                PSUB 480 PSUB0050
   30 DO 80 I=1,NDIM                                                    PSUB 490 PSUB0051
      IF (I-IDIMX) 40,40,60                                             PSUB 500 PSUB0052
   40 IF (I-IDIMY) 50,50,70                                             PSUB 510 PSUB0053
   50 Z(I)=X(I)-Y(I)                                                    PSUB 520 PSUB0054
      GO TO 80                                                          PSUB 530 PSUB0055
   60 Z(I)=-Y(I)                                                        PSUB 540 PSUB0056
      GO TO 80                                                          PSUB 550 PSUB0057
   70 Z(I)=X(I)                                                         PSUB 560 PSUB0058
   80 CONTINUE                                                          PSUB 570 PSUB0059
   90 IDIMZ=NDIM                                                        PSUB 580 PSUB0060
      RETURN                                                            PSUB 590 PSUB0061
      END                                                               PSUB 600 PSUB0062
```

```
      SUBROUTINE PADD(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PADD 390 PADD0001
C   THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE                           PADD0002
C       BEFORE COMPILING THIS UNDER  IBM FORTRAN G.                              PADD0003
C                                                                       PADD  10 PADD0004
C     ..................................................................PADD  20 PADD0005
C                                                                       PADD  30 PADD0006
C        SUBROUTINE PADD                                                PADD  40 PADD0007
C                                                                       PADD  50 PADD0008
C        PURPOSE                                                        PADD  60 PADD0009
C           ADD TWO POLYNOMIALS                                         PADD  70 PADD0010
C                                                                       PADD  80 PADD0011
C        USAGE                                                          PADD  90 PADD0012
C           CALL PADD(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PADD 100 PADD0013
C                                                                       PADD 110 PADD0014
C        DESCRIPTION OF PARAMETERS                                      PADD 120 PADD0015
C           Z     - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM      PADD 130 PADD0016
C                   SMALLEST TO LARGEST POWER                           PADD 140 PADD0017
C           IDIMZ - DIMENSION OF Z (CALCULATED)                         PADD 150 PADD0018
C           X     - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDEREDPADD 160 PADD0019
C                   FROM SMALLEST TO LARGEST POWER                      PADD 170 PADD0020
C           IDIMX - DIMENSION OF X (DEGREE IS IDIMX-1)                  PADD 180 PADD0021
C           Y     - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,       PADD 190 PADD0022
C                   ORDERED FROM SMALLEST TO LARGEST POWER              PADD 200 PADD0023
C           IDIMY - DIMENSION OF Y (DEGREE IS IDIMY-1)                  PADD 210 PADD0024
C                                                                       PADD 220 PADD0025
C        REMARKS                                                        PADD 230 PADD0026
C           VECTOR Z MAY BE IN SAME LOCATION AS EITHER VECTOR X OR      PADD 240 PADD0027
C           VECTOR Y ONLY IF THE DIMENSION OF THAT VECTOR IS NOT LESS   PADD 250 PADD0028
C           THAN THE OTHER INPUT VECTOR                                 PADD 260 PADD0029
C           THE RESULTANT POLYNOMIAL MAY HAVE TRAILING ZERO COEFFICIENTSPADD 270 PADD0030
C                                                                       PADD 280 PADD0031
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  PADD 290 PADD0032
C           NONE                                                        PADD 300 PADD0033
C                                                                       PADD 310 PADD0034
C        METHOD                                                         PADD 320 PADD0035
C           DIMENSION OF RESULTANT VECTOR IDIMZ IS CALCULATED AS THE    PADD 330 PADD0036
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C           LARGER OF THE TWO INPUT VECTOR DIMENSIONS. CORRESPONDING     PADD 340 PADD0037
C           COEFFICIENTS ARE THEN ADDED TO FORM Z.                        PADD 350 PADD0038
C                                                                         PADD 360 PADD0039
C     .................................................................. PADD 370 PADD0040
C                                                                         PADD 380 PADD0041
      DIMENSION Z(1),X(1),Y(1)                                            PADD 400 PADD0042
C                                                                         PADD 410 PADD0043
C     TEST DIMENSIONS OF SUMMANDS                                         PADD 420 PADD0044
C                                                                         PADD 430 PADD0045
      NDIM=IDIMX                                                          PADD 440 PADD0046
      IF (IDIMX-IDIMY) 10,20,20                                           PADD 450 PADD0047
   10 NDIM=IDIMY                                                          PADD 460 PADD0048
   20 IF(NDIM) 90,90,30                                                   PADD 470 PADD0049
   30 DO 80 I=1,NDIM                                                      PADD 480 PADD0050
      IF(I-IDIMX) 40,40,60                                                PADD 490 PADD0051
   40 IF(I-IDIMY) 50,50,70                                                PADD 500 PADD0052
   50 Z(I)=X(I)+Y(I)                                                      PADD 510 PADD0053
      GO TO 80                                                            PADD 520 PADD0054
   60 Z(I)=Y(I)                                                           PADD 530 PADD0055
      GO TO 80                                                            PADD 540 PADD0056
   70 Z(I)=X(I)                                                           PADD 550 PADD0057
   80 CONTINUE                                                            PADD 560 PADD0058
   90 IDIMZ=NDIM                                                          PADD 570 PADD0059
      RETURN                                                              PADD 580 PADD0060
      END                                                                 PADD 590 PADD0061
```

```
      SUBROUTINE PMPY(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PMPY 380 PMPY0001
C   THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE                           PMPY0002
C        BEFORE COMPILING THIS UNDER  IBM FORTRAN G.                             PMPY0003
C                                                                       PMPY  10 PMPY0004
C     ..................................................................PMPY  20 PMPY0005
C                                                                       PMPY  30 PMPY0006
C        SUBROUTINE PMPY                                                PMPY  40 PMPY0007
C                                                                       PMPY  50 PMPY0008
C        PURPOSE                                                        PMPY  60 PMPY0009
C           MULTIPLY TWO POLYNOMIALS                                    PMPY  70 PMPY0010
C                                                                       PMPY  80 PMPY0011
C        USAGE                                                          PMPY  90 PMPY0012
C           CALL PMPY(Z,IDIMZ,X,IDIMX,Y,IDIMY)                          PMPY 100 PMPY0013
C                                                                       PMPY 110 PMPY0014
C        DESCRIPTION OF PARAMETERS                                      PMPY 120 PMPY0015
C           Z     - VECTOR OF RESULTANT COEFFICIENTS, ORDERED FROM      PMPY 130 PMPY0016
C                   SMALLEST TO LARGEST POWER                           PMPY 140 PMPY0017
C           IDIMZ - DIMENSION OF Z (CALCULATED)                         PMPY 150 PMPY0018
C           X     - VECTOR OF COEFFICIENTS FOR FIRST POLYNOMIAL, ORDEREDPMPY 160 PMPY0019
C                   FROM SMALLEST TO LARGEST POWER                      PMPY 170 PMPY0020
C           IDIMX - DIMENSION OF X (DEGREE IS IDIMX-1)                  PMPY 180 PMPY0021
C           Y     - VECTOR OF COEFFICIENTS FOR SECOND POLYNOMIAL,       PMPY 190 PMPY0022
C                   ORDERED FROM SMALLEST TO LARGEST POWER              PMPY 200 PMPY0023
C           IDIMY - DIMENSION OF Y (DEGREE IS IDIMY-1)                  PMPY 210 PMPY0024
C                                                                       PMPY 220 PMPY0025
C        REMARKS                                                        PMPY 230 PMPY0026
C           Z CANNOT BE IN THE SAME LOCATION AS X                       PMPY 240 PMPY0027
C           Z CANNOT BE IN THE SAME LOCATION AS Y                       PMPY 250 PMPY0028
C                                                                       PMPY 260 PMPY0029
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  PMPY 270 PMPY0030
C           NONE                                                        PMPY 280 PMPY0031
C                                                                       PMPY 290 PMPY0032
C        METHOD                                                         PMPY 300 PMPY0033
C           DIMENSION OF Z IS CALCULATED AS IDIMX+IDIMY-1               PMPY 310 PMPY0034
C           THE COEFFICIENTS OF Z ARE CALCULATED AS SUM OF PRODUCTS     PMPY 320 PMPY0035
C           OF COEFFICIENTS OF X AND Y , WHOSE EXPONENTS ADD UP TO THE  PMPY 330 PMPY0036
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C              CORRESPONDING EXPONENT OF Z.                                  PMPY 340 PMPY0037
C                                                                            PMPY 350 PMPY0038
C     .......................................................................PMPY 360 PMPY0039
C                                                                            PMPY 370 PMPY0040
      DIMENSION Z(1),X(1),Y(1)                                               PMPY 380 PMPY0041
C                                                                            PMPY 400 PMPY0042
      IF(IDIMX*IDIMY)10,10,20                                                PMPY 410 PMPY0043
   10 IDIMZ=0                                                                PMPY 420 PMPY0044
      GO TO 50                                                               PMPY 430 PMPY0045
   20 IDIMZ=IDIMX+IDIMY-1                                                    PMPY 440 PMPY0046
      DO 30 I=1,IDIMZ                                                        PMPY 450 PMPY0047
   30 Z(I)=0.                                                                PMPY 460 PMPY0048
      DO 40 I=1,IDIMX                                                        PMPY 470 PMPY0049
      DO 40 J=1,IDIMY                                                        PMPY 480 PMPY0050
      K=I+J-1                                                                PMPY 490 PMPY0051
   40 Z(K)=X(I)*Y(J)+Z(K)                                                    PMPY 500 PMPY0052
   50 RETURN                                                                 PMPY 510 PMPY0053
      END                                                                    PMPY 520 PMPY0054
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```
      SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)                      PLRT 450 PLRT0001
C   THE ABOVE CARD SHOULD BE PLACED IN PROPER SEQUENCE                           PLRT0002
C        BEFORE COMPILING THIS UNDER  IBM FORTRAN G.                             PLRT0003
C                                                                       PLRT  10 PLRT0004
C     ..................................................................PLRT  20 PLRT0005
C                                                                       PLRT  30 PLRT0006
C        SUBROUTINE POLRT                                               PLRT  40 PLRT0007
C                                                                       PLRT  50 PLRT0008
C        PURPOSE                                                        PLRT  60 PLRT0009
C           COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL    PLRT  70 PLRT0010
C                                                                       PLRT  80 PLRT0011
C        USAGE                                                          PLRT  90 PLRT0012
C           CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)                      PLRT 100 PLRT0013
C                                                                       PLRT 110 PLRT0014
C        DESCRIPTION OF PARAMETERS                                      PLRT 120 PLRT0015
C           XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL          PLRT 130 PLRT0016
C                 ORDERED FROM SMALLEST TO LARGEST POWER                PLRT 140 PLRT0017
C           COF  -WORKING VECTOR OF LENGTH M+1                          PLRT 150 PLRT0018
C           M    -ORDER OF POLYNOMIAL                                   PLRT 160 PLRT0019
C           ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS    PLRT 170 PLRT0020
C                 OF THE POLYNOMIAL                                     PLRT 180 PLRT0021
C           ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE           PLRT 190 PLRT0022
C                 CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL       PLRT 200 PLRT0023
C           IER  -ERROR CODE WHERE                                      PLRT 210 PLRT0024
C                 IER=0  NO ERROR                                       PLRT 220 PLRT0025
C                 IER=1  M LESS THAN ONE                                PLRT 230 PLRT0026
C                 IER=2  M GREATER THAN 36                              PLRT 240 PLRT0027
C                 IER=3  UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS  PLRT 250 PLRT0028
C                        ON 5 STARTING VALUES                           PLRT 260 PLRT0029
C                 IER=4  HIGH ORDER COEFFICIENT IS ZERO                 PLRT 270 PLRT0030
C                                                                       PLRT 280 PLRT0031
C        REMARKS                                                        PLRT 290 PLRT0032
C           LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.                   PLRT 300 PLRT0033
C           FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER            PLRT 310 PLRT0034
C           POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULTS.PLRT 320 PLRT0035
C                                                                       PLRT 330 PLRT0036
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C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS
C        ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
C        RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
C        ERRORS IN THE REDUCED POLYNOMIAL.
C
C     ..................................................................
C
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     1 DX,DY,TEMP,ALPHA
C
C        ...............................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
      DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C        THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THE
C        CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO
C        1.0D-10.  THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE
C        COST OF EXECUTION TIME
C
C        ...............................................................
C
      IFIT=0
      N=M
      IER=0
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      IF(XCOF(N+1))10,25,10                                             PLRT 710 PLRT0073
   10 IF(N) 15,15,32                                                    PLRT 720 PLRT0074
C                                                                       PLRT 730 PLRT0075
C         SET ERROR CODE TO 1                                           PLRT 740 PLRT0076
C                                                                       PLRT 750 PLRT0077
   15 IER=1                                                             PLRT 760 PLRT0078
   20 RETURN                                                            PLRT 770 PLRT0079
C                                                                       PLRT 780 PLRT0080
C         SET ERROR CODE TO 4                                           PLRT 790 PLRT0081
C                                                                       PLRT 800 PLRT0082
   25 IER=4                                                             PLRT 810 PLRT0083
      GO TO 20                                                          PLRT 820 PLRT0084
C                                                                       PLRT 830 PLRT0085
C         SET ERROR CODE TO 2                                           PLRT 840 PLRT0086
C                                                                       PLRT 850 PLRT0087
   30 IER=2                                                             PLRT 860 PLRT0088
      GO TO 20                                                          PLRT 870 PLRT0089
   32 IF(N-36) 35,35,30                                                 PLRT 880 PLRT0090
   35 NX=N                                                              PLRT 890 PLRT0091
      NXX=N+1                                                           PLRT 900 PLRT0092
      N2=1                                                              PLRT 910 PLRT0093
      KJ1 = N+1                                                         PLRT 920 PLRT0094
      DO 40 L=1,KJ1                                                     PLRT 930 PLRT0095
      MT=KJ1-L+1                                                        PLRT 940 PLRT0096
   40 COF(MT)=XCOF(L)                                                   PLRT 950 PLRT0097
C                                                                       PLRT 960 PLRT0098
C         SET INITIAL VALUES                                            PLRT 970 PLRT0099
C                                                                       PLRT 980 PLRT0100
   45 XO=.00500101                                                      PLRT 990 PLRT0101
      YO=0.01000101                                                     PLRT1000 PLRT0102
C                                                                       PLRT1010 PLRT0103
C         ZERO INITIAL VALUE COUNTER                                    PLRT1020 PLRT0104
C                                                                       PLRT1030 PLRT0105
      IN=0                                                              PLRT1040 PLRT0106
   50 X=XO                                                              PLRT1050 PLRT0107
C                                                                       PLRT1060 PLRT0108
```

```
C          INCREMENT INITIAL VALUES AND COUNTER                                   PLRT1070 PLRT0109
C                                                                                 PLRT1080 PLRT0110
      X0=-10.0*Y0                                                                 PLRT1090 PLRT0111
      Y0=-10.0*X                                                                  PLRT1100 PLRT0112
C                                                                                 PLRT1110 PLRT0113
C          SET X AND Y TO CURRENT VALUE                                           PLRT1120 PLRT0114
C                                                                                 PLRT1130 PLRT0115
      X=X0                                                                        PLRT1140 PLRT0116
      Y=Y0                                                                        PLRT1150 PLRT0117
      IN=IN+1                                                                     PLRT1160 PLRT0118
      GO TO 59                                                                    PLRT1170 PLRT0119
   55 IFIT=1                                                                      PLRT1180 PLRT0120
      XPR=X                                                                       PLRT1190 PLRT0121
      YPR=Y                                                                       PLRT1200 PLRT0122
C                                                                                 PLRT1210 PLRT0123
C          EVALUATE POLYNOMIAL AND DERIVATIVES                                    PLRT1220 PLRT0124
C                                                                                 PLRT1230 PLRT0125
   59 ICT=0                                                                       PLRT1240 PLRT0126
   60 UX=0.0                                                                      PLRT1250 PLRT0127
      UY=0.0                                                                      PLRT1260 PLRT0128
      V =0.0                                                                      PLRT1270 PLRT0129
      YT=0.0                                                                      PLRT1280 PLRT0130
      XT=1.0                                                                      PLRT1290 PLRT0131
      U=COF(N+1)                                                                  PLRT1300 PLRT0132
      IF(U) 65,130,65                                                             PLRT1310 PLRT0133
   65 DO 70 I=1,N                                                                 PLRT1320 PLRT0134
      L =N-I+1                                                                    PLRT1330 PLRT0135
      TEMP=COF(L)                                                                 PLRT1340 PLRT0136
      XT2=X*XT-Y*YT                                                               PLRT1350 PLRT0137
      YT2=X*YT+Y*XT                                                               PLRT1360 PLRT0138
      U=U+TEMP*XT2                                                                PLRT1370 PLRT0139
      V=V+TEMP*YT2                                                                PLRT1380 PLRT0140
      FI=I                                                                        PLRT1390 PLRT0141
      UX=UX+FI*XT*TEMP                                                            PLRT1400 PLRT0142
      UY=UY-FI*YT*TEMP                                                            PLRT1410 PLRT0143
      XT=XT2                                                                      PLRT1420 PLRT0144
```

```
   70 YT=YT2                                                          PLRT1430 PLRT0145
      SUMSQ=UX*UX+UY*UY                                               PLRT1440 PLRT0146
      IF(SUMSQ) 75,110,75                                             PLRT1450 PLRT0147
   75 DX=(V*UY-U*UX)/SUMSQ                                            PLRT1460 PLRT0148
      X=X+DX                                                          PLRT1470 PLRT0149
      DY=-(U*UY+V*UX)/SUMSQ                                           PLRT1480 PLRT0150
      Y=Y+DY                                                          PLRT1490 PLRT0151
   78 IF(DABS(DY)+DABS(DX)-1.0D-05) 100,80,80                         PLRT1500 PLRT0152
C                                                                     PLRT1510 PLRT0153
C        STEP ITERATION COUNTER                                       PLRT1520 PLRT0154
C                                                                     PLRT1530 PLRT0155
   80 ICT=ICT+1                                                       PLRT1540 PLRT0156
      IF(ICT-500) 60,85,85                                            PLRT1550 PLRT0157
   85 IF(IFIT)100,90,100                                              PLRT1560 PLRT0158
   90 IF(IN-5) 50,95,95                                               PLRT1570 PLRT0159
C                                                                     PLRT1580 PLRT0160
C        SET ERROR CODE TO 3                                          PLRT1590 PLRT0161
C                                                                     PLRT1600 PLRT0162
   95 IER=3                                                           PLRT1610 PLRT0163
      GO TO 20                                                        PLRT1620 PLRT0164
  100 DO 105 L=1,NXX                                                  PLRT1630 PLRT0165
      MT=KJ1-L+1                                                      PLRT1640 PLRT0166
      TEMP=XCOF(MT)                                                   PLRT1650 PLRT0167
      XCOF(MT)=COF(L)                                                 PLRT1660 PLRT0168
  105 COF(L)=TEMP                                                     PLRT1670 PLRT0169
      ITEMP=N                                                         PLRT1680 PLRT0170
      N=NX                                                            PLRT1690 PLRT0171
      NX=ITEMP                                                        PLRT1700 PLRT0172
      IF(IFIT) 120,55,120                                             PLRT1710 PLRT0173
  110 IF(IFIT) 115,50,115                                             PLRT1720 PLRT0174
  115 X=XPR                                                           PLRT1730 PLRT0175
      Y=YPR                                                           PLRT1740 PLRT0176
  120 IFIT=0                                                          PLRT1750 PLRT0177
  122 IF(DABS(Y)-1.0D-4*DABS(X)) 135,125,125                          PLRT1760 PLRT0178
  125 ALPHA=X+X                                                       PLRT1770 PLRT0179
      SUMSQ=X*X+Y*Y                                                   PLRT1780 PLRT0180
```

```
      N=N-2
      GO TO 140
  130 X=0.0
      NX=NX-1
      NXX=NXX-1
  135 Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
  140 COF(2)=COF(2)+ALPHA*COF(1)
  145 DO 150 L=2,N
  150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
  155 ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF(SUMSQ) 160,165,160
  160 Y=-Y
      SUMSQ=0.0
      GO TO 155
  165 IF(N) 20,20,45
      END
```

V. SAMPLE RESULTS

	SHIP A	SHIP B
LBP=	528.50	475.00
BEAM=	76.00	68.31
DRAFT=	29.75	26.74
DISPL=	20900.00	15182.00
CB=	0.6125	0.6125
CP=	0.6246	0.6246
CM=	0.9807	0.9807

	SHIP A	SHIP B
DX/DU	-0.8400E 04	-0.6785E 04
DX/DUDOT	-0.1234E 07	-0.8960E 06
DY/DV	-0.8123E 05	-0.6561E 05
DY/DVDOT	-0.2271E 07	-0.1649E 07
DY/DPDOT	-0.1846E 08	-0.1340E 08
DY/DPDDOT	0.6677E 07	0.4357E 07
DY/DDEL	0.4898E 06	0.3957E 06
DN/DV	-0.9748E 07	-0.7077E 07
DN/DVDOT	0.1763E 08	0.1150E 08
DN/DPDOT	-0.3246E 10	-0.2118E 10
DN/DPDDOT	-0.3402E 11	-0.1995E 11
DN/DDEL	-0.1293E 09	-0.9387E 08
DX/DN	0.1710E 06	0.1242E 06
DY/DN	-0.1937E 05	-0.1406E 05
DN/DN	0.5113E 07	0.4595E 07

AT ALFA= 0.0 STSD= 60.0 AND BETA= 132.2
THE POSITION DEPENDENT DERIVATIVES ARE

	SHIP A	SHIP B
DY/DA	0.5399E 00	0.4853E 00
DY/DB	-0.2918E 01	0.2623E 01
DN/DA	0.2230E 03	0.1801E 03
DN/DB	0.7403E 03	-0.5930E 03
DY/DU	0.1067E 05	-0.8623E 04
DN/DU	-0.2432E 07	0.1802E 07

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  C.C            0.0            0.0
  0.0            0.0            C.0

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0            0.0            C.0
  0.0            0.0            0.0

SHIP A TIME LAGS ARE       0.0            C.0
SHIP B TIME LAGS ARE       0.0            0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
             0.0
             0.0
             C.1255E 32
             C.5102E 34
             C.9221E 36
             C.1460E 39
             0.2186E 41
             0.1949E 43
             C.7658E 44
             0.3479E 45
             C.2802E 46
PCLRT TEST VALUE IER=    0

CHAR. EQN. REAL ROOTS ARE
             0.0
             C.0
            -0.1250E-01
            -0.8846E-02
            -C.6807E-02
            -0.1280E 00
            -0.7573E-02
             0.1730E-02
             0.1730E-02
            -C.1424E 00
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE       0.0             0.0
SHIP B TIME LAGS ARE       0.0             0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
              0.1520E 34
              0.3168E 36
              0.1883E 38
              0.6835E 39
              0.1623E 41
              0.2645E 42
              0.2997E 43
              0.2399E 44
              0.1652E 45
              0.9746E 45
              0.2802E 46
POLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
             -0.6400E-01
             -0.7190E-02
             -0.1149E 00
             -0.1745E-01
             -0.1745E-01
             -0.3131E-01
             -0.3131E-01
             -0.1291E 00
              0.3243E-01
              0.3243E-01
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE       2.00          4.00
SHIP B TIME LAGS ARE       2.00          4.00


 THE DETERMINANT IN ASCENDING D
 POWERS IS
              0.1520E 34
              0.3046E 36
              0.1636E 38
              0.5489E 39
              0.1209E 41
              0.1821E 42
              0.1936E 43
              0.1497E 44
              0.1084E 45
              0.7140E 45
              0.2221E 46
PCLRT TEST VALUE IER=      0

CHAR. EQN. REAL ROOTS ARE
             -0.6552E-01
             -0.7190E-02
             -0.3311E-01
             -0.3311E-01
             -0.1124E 00
             -0.1631E-01
             -0.1631E-01
             -0.1261E 00
              0.4432E-01
              0.4432E-01


AT ALFA=      0.0 STSD=     90.0 AND BETA=    162.2
 THE POSITION DEPENDENT DERIVATIVES ARE

               SHIP A                  SHIP B
DY/DA          0.2733E 00              0.2456E 00
DY/DB         -0.1918E 01              0.1724E 01
DN/DA          0.1066E 03              0.8611E 02
DN/DB          0.4759E 03             -0.3844E 03
DY/DU          0.7685E 04             -0.6208E 04
DN/DU         -0.1715E 07              0.1245E 07
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0          0.0          0.0
  0.0          0.0          0.0

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0          0.0          0.0
  0.0          0.0          0.0

SHIP A TIME LAGS ARE      0.0            0.0
SHIP B TIME LAGS ARE      0.0            0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
             0.0
             0.0
             0.8124E 31
             0.3302E 34
             0.6707E 36
             0.1329E 39
             0.2171E 41
             0.1949E 43
             0.7658E 44
             0.8479E 45
             0.2802E 46
PCLRT TEST VALUE IER=     0

CHAR. FCN. REAL ROOTS ARE
             0.0
             0.0
            -0.1165E-01
            -0.8832E-02
            -0.6808E-02
            -0.1280E 00
            -0.7571E-02
             0.1296E-02
             0.1296E-02
            -0.1424E 00
```

```
SHIP A TIME LAGS ARE      0.0          0.0
SHIP B TIME LAGS ARE      0.0          0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
               0.1519E 34
               0.3167E 36
               0.1883E 38
               0.6833E 39
               0.1622E 41
               0.2645E 42
               0.2996E 43
               0.2399E 44
               0.1652E 45
               0.9746E 45
               0.2802E 46
PCLRT TEST VALUE IER=     0

CHAR. FCN. REAL ROOTS ARE
              -0.6399E-01
              -0.7190E-02
              -0.1149E 00
              -0.1745E-01
              -0.1745E-01
              -0.3131E-01
              -0.3131E-01
              -0.1291E 00
               0.3243E-01
               0.3243E-01
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000       4.000000      -0.010000
 -0.100000       0.010000       0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000       4.000000       0.010000
  0.100000      -0.010000      -0.200000

SHIP A TIME LAGS ARE        2.00              4.00
SHIP B TIME LAGS ARE        2.00              4.00


 THE DETERMINANT IN ASCENDING D
 POWERS IS
             0.1519E 34
             0.3046E 36
             0.1636E 38
             0.5488E 39
             0.1208E 41
             0.1821E 42
             0.1936E 43
             0.1497E 44
             0.1084E 45
             0.7140E 45
             0.2221E 46
POLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
            -0.6552E-01
            -0.7190E-02
            -0.3311E-01
            -0.3311E-01
            -0.1124E 00
            -0.1631E-01
            -0.1631E-01
            -0.1261E 00
             0.4431E-01
             0.4431E-01


AT ALFA=   200.0 STSD=    70.0 AND BETA=    142.2
 THE POSITION DEPENDENT DERIVATIVES ARE

                SHIP A                    SHIP B
DY/DA        -0.6247E 00               0.5186E 00
DY/DB        -0.6854E-01              -0.1170E 01
DN/DA         0.3876E 02              -0.2561E 03
DN/DB        -0.2293E 03              -0.4629E 02
DY/DU         0.2351E 04               0.1942E 04
DN/DU         0.8342E 06               0.2333E 06
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0            0.0            0.0
  0.0            0.0            0.0

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0            0.0            0.0
  0.0            0.0            0.0

SHIP A TIME LAGS ARE       0.0            0.0
SHIP B TIME LAGS ARE       0.0            0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
              0.0
              0.0
             -0.1766E 31
             -0.7038E 33
              0.1157E 36
              0.1044E 39
              0.2141E 41
              0.1949E 43
              0.7659E 44
              0.8479E 45
              0.2802E 46
POLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
              0.0
              0.0
             -0.9162E-02
             -0.7577E-02
             -0.6806E-02
             -0.1279E 00
             -0.7340E-02
              0.2534E-02
             -0.1424E 00
             -0.3938E-02
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE       0.0             0.0
SHIP B TIME LAGS ARE       0.0             0.0


 THE DETERMINANT IN ASCENDING D
 POWERS IS
              0.1519E 34
              0.3166E 36
              0.1882E 38
              0.6831E 39
              0.1622E 41
              0.2644E 42
              0.2996E 43
              0.2399E 44
              0.1652E 45
              0.9746E 45
              0.2802E 46
PCLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
             -0.6397E-01
             -0.7190E-02
             -0.1149E 00
             -0.1745E-01
             -0.1745E-01
             -0.3131E-01
             -0.3131E-01
             -0.1291E 00
              0.3241E-01
              0.3241E-01
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE        2.00            4.00
SHIP B TIME LAGS ARE        2.00            4.00


 THE DETERMINANT IN ASCENDING D
 POWERS IS
                 0.1519E 34
                 0.3045E 36
                 0.1635E 38
                 0.5486E 39
                 0.1208E 41
                 0.1820E 42
                 0.1936E 43
                 0.1497E 44
                 0.1085E 45
                 0.7140E 45
                 0.2221E 46
POLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
                -0.6550E-01
                -0.7190E-02
                -0.3311E-01
                -0.3311E-01
                -0.1124E 00
                -0.1631E-01
                -0.1631E-01
                -0.1261E 00
                 0.4430E-01
                 0.4430E-01


AT ALFA=  -400.0 STSD=     30.0 AND BETA=    152.2
 THE POSITION DEPENDENT DERIVATIVES ARE

                SHIP A                  SHIP B
DY/DA          0.1457E 00            -0.2734E 00
DY/DB          0.1492E 01            -0.3327E 00
DN/DA         -0.1208E 03            -0.4958E 02
DN/DB         -0.2523E 03             0.1403E 03
DY/DU         -0.3333E 04            -0.2426E 03
DN/DU          0.6393E 06            -0.4488E 06
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0           0.0           0.0
  0.0           0.0           0.0

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  0.0           0.0           0.0
  0.0           0.0           0.0

SHIP A TIME LAGS ARE      0.0           0.0
SHIP B TIME LAGS ARE      0.0           0.0


 THE DETERMINANT IN ASCENDING O
 POWERS IS
             0.0
             0.0
            -0.3609E 31
            -0.1458E 34
             0.8493E 34
             0.9850E 38
             0.2132E 41
             0.1948E 43
             0.7659E 44
             0.8479E 45
             0.2802E 46
ROUTH TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
             0.0
             0.0
            -0.9157E-02
            -0.7574E-02
            -0.6129E-02
            -0.6129E-02
            -0.1279E 00
             0.3502E-02
            -0.1424E 00
            -0.6806E-02
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE        0.0            0.0
SHIP B TIME LAGS ARE        0.0            0.0


 THE DETERMINANT IN ASCENDING O
 POWERS IS
               0.1519E 34
               0.3165E 36
               0.1882E 38
               0.6831E 39
               0.1622E 41
               0.2644E 42
               0.2996E 43
               0.2399E 44
               0.1652E 45
               0.9746E 45
               0.2802E 46
PCLRT TEST VALUE IER=     0

CHAR. EQN. REAL ROOTS ARE
              -0.6397E-01
              -0.7190E-02
              -0.1149E 00
              -0.1745E-01
              -0.1745E-01
              -0.3131E-01
              -0.3131E-01
              -0.1291E 00
               0.3241E-01
               0.3241E-01
```

```
SHIP A CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000     -0.010000
 -0.100000      0.010000      0.200000

SHIP B CONTROL SYSTEM CONSTANTS 1 TO 6
  1.000000      4.000000      0.010000
  0.100000     -0.010000     -0.200000

SHIP A TIME LAGS ARE       2.00            4.00
SHIP B TIME LAGS ARE       2.00            4.00


 THE DETERMINANT IN ASCENDING D
 POWERS IS
            0.1519E 34
            0.3044E 36
            0.1635E 38
            0.5486E 39
            0.1208E 41
            0.1820E 42
            0.1936E 43
            0.1497E 44
            0.1085E 45
            0.7140E 45
            0.2221E 46
FCLRT TEST VALUE IER=    0

CHAR. EQN. REAL ROOTS ARE
           -0.6550E-01
           -0.7190E-02
           -0.3311E-01
           -0.3311E-01
           -0.1124E 00
           -0.1631E-01
           -0.1631E-01
           -0.1261E 00
            0.4429E-01
            0.4429E-01
```